CONTRACTIBILITY OF CERTAIN SEMIGROUPS

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1. It has long been known that a compact, connected, triangulable topological group, G, must have its Euler-Poincaré number $\nu(G)=0$.

The analogous result is not true if G is replaced by S, a topological semigroup with identity. (Witness the unit interval under multiplication.) However, we will show: If S is a compact, connected, triangulable semigroup with identity, then $\nu(S)=0$ or $\nu(S)=1$ and, in the latter case, S is contractible.

2. We follow, in part, the terminology of [1].

Let S be any topological semigroup, then R denotes a minimal right ideal of S, if such exists.

Let us say that a semigroup S satisfies * if $x\in S$ implies there exists a $y\in S$ such that $xy=y$.

We recall that a space is contractible if the identity mapping is homotopic to a constant map.

LEMMA. *Let S be a compact, arcwise connected topological semigroup with identity element e. If S satisfies *, then S is contractible.*

PROOF. It is known [1] that S has a minimal right ideal R and that $a\in R$ implies $aR=R=aS$. Fix any $x\in R$, then by * above, there is a $y\in S$ such that $y=xy\in xS=R$. If $z\in R$, then there is a $z'\in R$ such that $yz'=z$; thus $xz=xyz'=yz'=z$. It follows that $xy=y$ for any $x, y\in R$.

Let $i\colon S\rightarrow S$ be the identity mapping and let $x\in R$. Since S is arcwise connected, there is an arc from e to x; let $p\colon I\rightarrow S$ be such an arc (I denotes the unit interval), with $p(0)=e$ and $p(1)=x$.

Define

$$h\colon S\times I\rightarrow S$$

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