by $h(s, t)=p(t)sp(t)$. Then $h(s, 0)=s=i(s)$ and $h(s, 1)=(xs)x=x$. Hence i is homotopic to a constant map and thus S is contractible.

REMARKS. It should be observed that in the proof above the arcwise connectedness of S can be replaced by the existence of an arc between e and some element of the minimal ideal of S.

The necessity of a two-sided identity is clearly indicated by the example of two tangent circles with multiplication $xy=y$.

THEOREM. *Let S be a compact, arcwise connected, topological semigroup with identity e. If S has no fixed point free deformations, then S is contractible.*

PROOF. It is easy to see that multiplication by a fixed element of S is homotopic to the identity, thus it must have a fixed point and, hence, S satisfies $*$. It follows from the lemma that S is contractible.

COROLLARY. *Let S be compact and triangulable with nonzero Euler-Poincaré characteristic. If S is a topological semigroup with an identity then S is contractible.*

It need only be noted that S has no fixed point free deformations by a well-known corollary to the Lefschetz fixed-point theorem [2].

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