A NEW IN Variant of HOMOTOPY TYPE
AND SOME DIVERSE APPLICATIONS

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Let $X$ be a connected, locally finite simplicial polyhedron. Let $X^X$ be the space of maps from $X$ to $X$ with the compact-open topology. Let $x_0 \in X$ be taken as a base point in $X$, then the evaluation map $\rho: X^X \to X$ defined by $\rho(f) = f(x_0)$ for $f \in X^X$ is continuous. Now $\rho$ induces the homomorphism

$\rho_*: \pi_1(X^X, 1_x) \to \pi_1(X, x_0),$

where $1_x \in X^X$ is the identity map. Hence $\rho_*\pi_1(X^X, 1_x)$ is a subgroup of the fundamental group of $(X, x_0)$.

Proposition 1. $\rho_*\pi_1(X^X, 1_x)$ considered as a subgroup of $\pi_1(X, x_0)$ is an invariant of homotopy type.

In [2], this invariant is studied and theorems are obtained which bear on the study of $X^X$, groups of homeomorphisms, homological group theory and knot theory. Most of these results come from the following theorem.

Theorem 2. Let $X$ have the homotopy type of a compact, connected polyhedron with nonzero Euler-Poincaré number. Then $\rho_*\pi_1(X^X, 1_x) = 0$.

The proof of this employs Nielsen-Wecken fixed-point class theory ([1] and [5]).

Let $G(X)$ be the group of homeomorphisms of a manifold $X$, and let $G_0(X)$ be the isotropy group over $x_0$. Then there is an exact sequence [3]

$$\ldots \to \pi_i(G_0(X), 1_x) \to \pi_i(G(X), 1_x) \xrightarrow{\rho'_*} \pi_i(X, x_0) \to \ldots,$$

where $\rho': G(X) \to X$ is the evaluation map.

Corollary 3. Let $X$ be as in Theorem 2. Then $\rho'_*\pi_i(G(X), 1_x) = 0$. In particular, if $\pi_0(X, x_0) = 0$, then $i_*: \pi_1(G_0(X), 1_x) \to \pi_1(G(X), 1_x)$.

This follows because $\rho'_*\pi_1(G(X), 1_x) \subseteq \rho_*\pi_1(X^X, 1_x)$.

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