Theorem 4. If $X$ is an aspherical polyhedron, then $p\pi_1(X^x, 1_x) = Z(\pi_1(X, x_0))$, the center of $\pi_1(X, x_0)$.

Theorems 2 and 4 combine to give us the following corollaries:

Corollary 5. If $X$ has the same homotopy type as a compact, connected, aspherical polyhedron with nonzero Euler-Poincaré number, then $Z(\pi_1(X, x_0)) = 0$.

John Stallings, in [4], has put this result in a purely algebraic setting; namely, if a group $G$ admits a finite resolution, then, if $Z(G)$ is nontrivial, the (suitably defined) Euler-Poincaré number is zero.

Alexander’s Duality and the last corollary gives us a result suggested by L. P. Neuwirth.

Corollary 6. Suppose that $X$ is a subcomplex of the $n$-sphere $S^n$ whose Euler characteristic is different from that of $S^n$. If $S^n - X$ is connected and aspherical, then $\pi_1(S^n - X)$ has no center.

Finally, we are able to show the following:

Theorem 7. If $X$ is aspherical, then
\[
\pi_1(X^x, 1_x) \cong Z(\pi_1(X, x_0)),
\]
\[
\pi_n(X^x, 1_x) \cong 0, \quad n > 1.
\]

Note that Theorem 7 and Theorem 2 give us:

Corollary 8. If $X$ has the homotopy type of an aspherical compact polyhedron whose Euler characteristic is different from zero, then the identity component of $X^x$ is contractible.

Bibliography


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