A CERTAIN CLASS OF INCIDENCE MATRICES

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1. Let S be a set consisting of K elements, and call any subset of S containing precisely m elements an *m-set* [2]. We wish to study incidence matrices obtained in the following manner: Let $K \geqq m \geqq n \geqq 0$, we label the rows of the matrix by all the m-sets of S and the columns by all the n-sets of S; take the (i, j) element as 1 if the m-set corresponding to the ith row contains the n-set corresponding to the jth column, and zero otherwise.

This note studies the question of which collections of m-sets give rise to a linearly dependent set of row vectors, and likewise what combinations of n-sets give rise to a linearly dependent set of column vectors. To do this, we characterize the row null spaces and the column null spaces of the above matrices in an inductive manner. Then using this characterization, we prove that the above matrices must have maximal rank. A corollary then gives a necessary condition for the existence of a tactical configuration [1].

2. Now for any $K \geqq m \geqq n \geqq 0$, we shall define a canonical matrix $A^K_{m,n}$.

We first define a lexicographical ordering of the m-sets of $\{1, \cdots, K\}$, the set of integers from one to K, in the following way. Represent any m-set as $(a_1, \cdots, a_m)$ where $a_i < a_{i+1}$ for $i = 1, \cdots, m$. Then we say that $(a_1, \cdots, a_m) < (b_1, \cdots, b_m)$ if and only if $a_i < b_i$ for the smallest value of i for which $a_i \neq b_i$. We shall call this ordering the *canonical ordering* of the m-sets of $\{1, \cdots, K\}$.

We now define $A^K_{m,n}$ to be the incidence matrix whose rows correspond to the m-sets of $\{1, \cdots, K\}$ in their canonical order, and whose columns correspond to the n-sets in their canonical order. For example, the first row of $A^K_{m,n}$ corresponds to $(1, 2, \cdots, m)$ and the first column to $(1, 2, \cdots, n)$.

3. We gather some simple facts about $A^K_{m,n}$ here. $C_{a,b}$ will represent the usual binomial coefficient.

LEMMA 1. *$A^K_{m,n}$ has*

(a) $C_{K,m} = K!/m!(K-m)!$ *rows,*

(b) $C_{K,n}$ *columns,*

(c) $C_{m,n}$ *ones in each row,*

(d) $C_{K-n,m-n}$ *ones in each column.*

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