

$\dim(\text{RN}_{m,n}^K) = 0$. Hence the rank of the matrix must be the number of rows of $A_{m,n}^K$, which is $C_{K,m}$ (see Lemma 1). This implies that $\dim(\text{CN}_{m,n}^K) = \text{number of columns of } A_{m,n}^K - \text{number of rows} = C_{K,n} - C_{K,m}$. For the other case, when $m+n \leq K$, use equation (2) and the same argument as above.

COROLLARY 2. $A_{m,n}^K$ has maximal rank.

6. A tactical configuration $C[k, l, \lambda, v]$, [1], is defined as follows. Given a set E of v elements, and given positive integers k, l ($l \leq k \leq v$), a tactical configuration is a system of k -sets of E such that each l -set is contained in exactly λ k -sets of the system.

COROLLARY 3. If $C[k, l, \lambda, v]$ does not contain every k -set of E , then $k+l < v$.

PROOF. A tactical configuration $C[k, l, \lambda, v]$ corresponds to a set of rows, S , of $A_{k,l}^v$. Let r_i be a row vector. Then

$$(1) \quad \sum_{r_i \in S} r_i = (\lambda, \lambda, \dots, \lambda) = \lambda(1, \dots, 1).$$

On the other hand, letting $L = C_{v-l, k-l}$,

$$(2) \quad \sum_{\text{all } r_i} r_i = L(1, \dots, 1).$$

Hence we obtain

$$\sum_{r_i \in S} Lr_i - \sum_{\text{all } r_i} \lambda r_i = 0$$

or

$$\sum_{r_i \in S} (L - \lambda)r_i + \sum_{r_i \notin S} -\lambda r_i = 0.$$

Since S does not include all the rows of $A_{k,l}^v$, we see that the row vectors form a linearly dependent set. Hence, since $A_{k,l}^v$ has maximal rank, there are more rows than columns. That is, $C_{K,m} > C_{K,n}$, hence $m+n < K$.

BIBLIOGRAPHY

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