Peter Schlamiel tried to use Stokes' theorem

$$\oint_C \vec{F}\cdot d\vec{r} = \iint_S (\vec{\nabla}\times\vec{F})\cdot\vec{n}\; d\sigma$$

where S is the surface $\{(x,y,z)$ so that $x^2+y^2+4z^2=4$ and $y\le 1\}$ and C is the boundary closed curve on the surface given by $y=1$. Here $\vec{F} = \dfrac{x}{x^2+y^2}\;\vec{i} + \dfrac{y}{x^2+y^2}\;\vec{j} + x\vec{k}$.

Find Schlamiel's mistakes. One point for each error you find, minus one point for each correct thing you say is an error. Give reasons for your statements. Unclear or incorrect reasons will receive 0 or -1 points. Correct reasons will get $+1$ points each:

$$\vec{\nabla}\times\vec{F} = -\vec{j}$$

$$\vec{n} = 2x\vec{i} + 2y\vec{j} + 8z\vec{k}$$

$$(\vec{\nabla}\times\vec{F})\cdot\vec{n} = -2y.$$

Parameterize S by $\vec{r}(u,v) = v\cos u\vec{i} + v\sin u\vec{j} + \frac{1}{2}\sqrt{4-v^2}\;\vec{k}$ where $0\le u\le 2\pi$ and $-2\le v\le 2$.

$$\vec{r}_u = -v\sin u\vec{i} + v\cos u\vec{j}$$

$$\vec{r}_v = \cos u\vec{i} + \sin u\vec{j} - \frac{v}{2\sqrt{4-v^2}}\;\vec{k}$$

$$\vec{r}_u\times\vec{r}_v = \frac{-v^2\cos u}{2\sqrt{4-v^2}}\;\vec{i} + \left(\frac{-v^2\sin u}{2\sqrt{4-v^2}}\right)\vec{j} - v\vec{k}$$

$$d\sigma = \sqrt{\frac{v^4}{4(4-v^2)} + v^2}\; dudv$$

$$\iint_S (\vec{\nabla}\times\vec{F})\cdot\vec{n}d\sigma = \int_{-2}^{2}\int_0^{2\pi} (-2v\cos u)\sqrt{\frac{v^4}{16-4v^2}+v^2}\; du\; dv$$

Now $C = \{(x,y,z)|x^2+1+4z^2=4,\; y=1\}$. Let $\vec{r}(x)$ parametrize C where

$$\vec{r}(x) = x\vec{i} + \vec{j} + \frac{1}{2}\sqrt{3-x^2}\;\vec{k}, \qquad \text{where } -\sqrt{3}\le x\le\sqrt{3}$$

$$\frac{d\vec{r}}{dx} = (\vec{i} - \frac{x}{2\sqrt{3-x^2}}\;\vec{k})$$

$$\vec{F}\cdot\frac{d\vec{r}}{dx} = \left(\frac{x}{x^2+1}\;i + \frac{1}{x^2+1}\;j + x\vec{k}\right)\cdot\left(\vec{i} - \frac{x}{2\sqrt{3-x^2}}\;k\right)$$

$$\vec{F}\cdot\frac{dr}{dx} = \frac{x}{x^2+1} - \frac{x^2}{2\sqrt{3-x^2}}$$

$$\oint_C \vec{F}\cdot d\vec{r} = \int_{-\sqrt{3}}^{\sqrt{3}}\left(\frac{x}{x^2+1} - \frac{x^2}{2\sqrt{3-x^2}}\right)dx$$

So

$$\int_{-2}^{2}\int_0^{2\pi}(-2v\cos u)\sqrt{\frac{v^4}{16-4v^2}+v^2}\; dv\; du$$

$$= \int_{-\sqrt{3}}^{\sqrt{3}}\left(\frac{x}{x^2+1} - \frac{x^2}{2\sqrt{3-x^2}}\right)dx$$