Math 174 Exam II 3/31/2000 D. Gottlieb

(5) 1. Suppose that $f(x, y)$ is a real valued function defined everywhere on the plane. Suppose its gradient $\overrightarrow{\nabla f}$ is not defined at $(0, 0)$.

a) Is it possible to determine whether or not f is continuous at $(0, 0)$ given the above information? (if yes, is it continuous at $(0, 0)$ or not)

b) Is it possible to determine whether or not $f(x, y)$ is differentiable at $(0, 0)$? (if yes, is it differentiable at $(0, 0)$)

HINT: If you don't know the answers for a) or b), write down the definitions of continuity for a) or differentiability for b) for some partial credit.

(20) 2. Let $f(x, y, z) = xyz$ and P be the point $(2, -3, 4)$.

a) Calculate the gradient vector field $\overrightarrow{\nabla f}$.

b) Find the gradient vector of f at the point P $(2, -3, 4)$.

c) Find the directional derivative $(D_{\vec{u}} f)_P$ at the point P $(2, -3, 4)$ in the direction $\vec{u} = \frac{1}{\sqrt{2}}\,\vec{i} - \frac{1}{\sqrt{2}}\,\vec{k}$.

d) In which direction $\vec{v}$ does the directional derivative $(D_{\vec{v}} f)_p$ have its minimum value at the point P.

e) Give the equation of the level surface of $f(x, y, z)$ through the point P.

f) Find the normal vector to the level surface at P.

g) Find the tangent plane to the level surface at P.

h) Give a critical point of $f(x, y, z)$ and state whether or not it gives a maximum or minimum value of $f(x, y, z)$.

(10) 3. Let $w = f(x, y, z)$ be a differentiable function and x, y, z be differentiable functions of t.

a) Express $\frac{dw}{dt}$ by the chain rule.

b) Express $\frac{dw}{dt}$ as the dot product of a gradient vector field and a velocity vector field of a path.

c) What is the rate of change with respect to t of $f(x, y, z) = x + y^2 + z^3$ along the path $t\vec{i} + t\vec{j} + 2\vec{k}$ at the point $(0, 0, 2)$?

(10) 4. Consider the curve defined by the intersection of the surfaces $x^2 + y^2 + z^2 = 2$ and $x^2 + 2y^2 = 1$. Find a tangent vector $\vec{T}$ for this curve at the point $(1, 0, 1)$.

(5) 5.a) Write down the second order (quadratic) Taylor's formula for the function $f(x, y)$ about $(0, 0)$.

b) Suppose $f(0, 0) = 0$, $\vec{\nabla} f|_{(0,0)} = \vec{0}$ and the second order partial derivatives are all equal to 3 at $(0, 0)$. Write down the second order Taylor's formula in this case.

(50) 6. Find the volume under the paraboloid $z = x^2 + y^2$ and above the triangle in the xy plane enclosed by the lines $y = x$ and $x + y = z$ and the y–axis.

a) Sketch the region of integration.

b) Set up the integral.

c) Evaluate the integral.