Math 174 Exam II 3/5/99 D. Gottlieb

Name ____________________________________

1. a) Give an example of a set in the plane which is open

AND

b) Give an example of a set which is closed in the plane.

2. Do one of the two questions.

a) A function $f(x, y)$ is continuous at the point (x_0, y_0) if what three conditions are satisfied?

OR

b) Consider the function $\begin{cases} f(x,y) = \dfrac{x^4}{x^4 + y^2} & \text{for } x \text{ and } y \text{ not both zero. Is this} \\ f(0,0) = 0. \end{cases}$ function continuous at $(0, 0)$?

3. a) Suppose $f(x, y)$ and all partial derivatives are defined in an open neighborhood of a point (a, b). What does Euler's theorem say about f_{xy}?

OR

b) Let $f(x, y) = xy^3 e^x \cos y + y^4 \ln(x)$. What number does $f_{xx}(f_{xy} - f_{yx})$ equal when evaluated at $(\pi/2, 1)$?

4. a) Suppose $f(x, y, z)$ is a differentiable function so that $f_x(a, b, c) = 2,\ f_y(a, b, c) = -1$ and $f_z(a, b, c) = 0$. If the values of b and c are changed by $dy = 1$ and $dz = .1$, what change dx of a will result in $df = 0$?

OR

b) Let $f(x, y, z) = (e^{x+y})z + \sqrt{x^2 + y^2} \ln z$. Given $dy = .1,\ dz = .1$ and $df = 0$ at the point $(3, 2, 1)$, what value must dx have?

5. Suppose $f(x, y, z) = x^2 + y^2 + z^2$ and $g(x, y, z) = x + y + z$. Sketch the level surfaces $f(x, y, z) = f(1, -1, 1)$ and $g(x, y, z) = g(1, -1, 1)$.

6. Find the gradient vectors of $f(x, y, z) = x^2 + y^2 + z^2$ and $g(x, y, z) = x + y + z$ at the point $(1, -1, 1)$ and find a vector perpendicular to both of them.

7. a) Which function $f(x, y, z) = x^2 + y^2 + z^2$, or $g(x, y, z) = x + y + z$ has the fastest rate of increase at $(1, -1, 1)$.

AND

b) What is the tangent vector to the curve of intersection at $(1, -1, 1)$?

8. At what point is the maximum value of $g(x, y, z) = x + y + z$ subject to the constraint $f(x, y, z) = x^2 + y^2 + z^2 = 3$ taken on?

9. Find the equation of the tangent plane to the sphere $x^2 + y^2 + z^2 = \frac{1}{3}$ at the point $\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$.

10. Use Taylor's formula for $f(x, y) = e^x \cos y$ to find a quadratic approximation near the origin.