Math 174 | Exam III | April 9, 1999 | D. Gottlieb

Name ________________________________

1. a) Give the definition of the triple integral of a function $f(x,y,z)$ over a bounded region of space.

OR

   b) Evaluate $\int_1^3 \int_1^3 \int_1^3 \frac{1}{xyz}\, dxdydz$.

2. a) Find the average height of the hemisphere $z = \sqrt{a^2 - x^2 - y^2}$ over the disk $x^2 + y^2 \leq a$.

OR

   b) The average value of function $f(x,y)$ equals $3\pi r$ over a disk D of radius r in the plane. What is the value of $\iint_D f(x,y)dA$?

3. The area of a region $R = \int_0^1 \int_{1-x}^{1-x^2} dydx$. Sketch R and set up the integral of the area of R with the order of integration reversed.

4. Set up the triple integrals for the volume of the sphere of radius 5 and center at the origin using
   a) rectangular coordinates
   b) cylindrical coordinates
   c) spherical coordinates
   d) By the way, what is the volume of that sphere? (You do not have to integrate if you already know the answer).

5. a) Solve the system of equations
$u = 3x + 2y,\ \ v = x + 4y$ for x and y in terms of u and v. Then find the Jacobian $\partial(x, y)/\partial(u, v)$.

b) Let ABC be a triangle in the xy plane where $A = (0, 0),\ B = (1, 0),\ C = (0, 1)$. Let $T\colon (xy\text{–plane}) \longrightarrow (uv\text{–plane})$ be the transformation T given by $(x, y) = (3x + 2y,\ x + 4y)$. What is the image of the triangle under T in the uv plane?

6. Consider the L shaped region of width 1 in.

a) Find its centroid $(\overline{x}, \overline{y})$.

OR

b) Find its center of mass $\vec{c}$ when the L has constant density by using Pappus's formula $\vec{c} = \dfrac{1}{m_1 + m_2}\,(\vec{c}_1 + \vec{c}_2)$ where $\vec{c}_1$ is the center of mass of a body of mass m_1 and $\vec{c}_2$ is the center of mass of a body of mass m_2 which does not overlap with body 1. (You are not forced to integrate in this problem.)