MA 490G Exam 1 Spring 2000 D. Gottlieb

1. Let $f\colon X \to Y$ and $g\colon Y \to Z$. Suppose $g \circ f\colon X \to Z$ is a bijection,
   a) Show that g is onto and f is $1-1$.
   b) Give an example where g is not injective and f is not surjective.

2. A Hausdorff space is a topological space so that any two different points have disjoint open neighborhoods.
   a) Show that points are closed sets in a Hausdorff space.
   b) Give an example of a space which is not Hausdorff.
   c) Show a fibre of any continuous map $f\colon X \to Y$ is a closed set if Y is a Hausdorff space.
   d) Show that a subspace of a Hausdorff space is a Hausdorff space.
   e) Show the product of a Hausdorff space with another Hausdorff space is Hausdorff.

3. Show that a product space $X_1 \times \ldots \times X_n$ is connected if and only if $X_1, \ldots, X_n$ are all connected.