(8) 1. Let $G$ be a finite group.
   (i) What is meant by a composition series for $G$?
   (ii) State the Jordan-Hölder Theorem.

(9) 2. (i) Diagram the lattice of subgroups of the dihedral group $D_8$.
   (ii) How many different composition series exist for the dihedral group $D_8$?
3. Let $G$ be a finite group and let $P \in \text{Syl}_p(G)$ be a Sylow $p$-subgroup of $G$. If $Q$ is a $p$-subgroup of $G$, prove that $Q \cap N_G(P) = Q \cap P$.

4. List all positive integers that are the order of an element of the symmetric group $S_8$.

5. For $n$ a positive integer, let $Z_n$ denote a cyclic group of order $n$.
   (i) Are the groups $\text{Aut}(Z_7)$ and $\text{Aut}(Z_9)$ isomorphic? Justify your answer.
   (ii) Are the groups $\text{Aut}(Z_8)$ and $\text{Aut}(Z_{10})$ isomorphic? Justify your answer.
6. Let $G$ be a group of order 18. Prove that $G$ has a subgroup of order 9.

7. Let $H$ be the cyclic subgroup of $S_4$ generated by the 4-cycle $(1\ 2\ 3\ 4)$.
   
   (i) What is the order of the normalizer $N$ of $H$ in $S_4$?

   (ii) Give generators for the group $N$. 
8. Assume that $G_1$ and $G_2$ are groups and that $N_i$ is a normal subgroup of $G_i$, $i = 1, 2$. If $G_1 \cong G_2$ and $G_1/N_1 \cong G_2/N_2$, prove or disprove that $N_1$ is isomorphic to $N_2$.

9. Recall that a subgroup $H$ of a group $G$ is called a characteristic subgroup if $\phi(H) = H$ for every automorphism $\phi$ of $G$. Give an example of a group $G$ and a normal subgroup $N$ of $G$ such that $N$ is not a characteristic subgroup of $G$. Explain why in your example $N$ is normal but not characteristic.
(6) 10. Give an example of a commutative ring $R$ with identity $1 \neq 0$ that has ideals $I$ and $J$ such that $\{ab \mid a \in I, b \in J\}$ is not an ideal of $R$.

(6) 11. Diagram the lattice of ideals of the ring $\mathbb{Z}/(30)$.

(6) 12. Diagram the lattice of ideals of the ring $\mathbb{Z}/(36)$. 
13. Let $R$ be a commutative ring with 1.

(i) Define the characteristic of $R$.

(ii) Does there exist a ring having characteristic 4? Justify your answer.

14. If $R$ is an integral domain, prove that the polynomial ring $R[x]$ has no zero divisors.