Math 373
Quiz 4
Fall 2018
October 25, 2018

1. Jake makes deposits at the start of each quarter for ten years into the Rahn Fund. The Rahn Fund pays an annual effective interest rate of 6%.

   The payments are P at the beginning of each quarter in the first year. The payments are $2P$ at the beginning of each quarter in the second year. The payments continue to increase each year until payments of $10P$ are paid at the beginning of each quarter in the tenth year.

   At the end of 10 years, Jake has 100,000.

   Determine P.

   Solution:

   We start by noting that this is an annuity due and that we need the accumulated value. We also note that since the payments are level for each year, we need to use the formula that does not follow the rules.

   $$1+i = \left(1+\frac{i^{(4)}}{4}\right)^4 \implies 1.06 = \left(1+\frac{i^{(4)}}{4}\right)^4 \implies \frac{i^{(4)}}{4} = (1.06)^{0.25} - 1 = 0.014673846$$

   $$P\left[\frac{\ddot{a}_{\overline{10}|0.06} - 10(1.06)^{-10}}{0.014673846}\right](1.014673846)(1.06)^{10} = 100,000$$

   $$\ddot{a}_{\overline{10}|0.06} = \frac{1-(1.06)^{-10}}{0.06}(1.06) = 7.801692273$$

   $$P = \frac{100,000}{\left[\dfrac{7.801692273 - 10(1.06)^{-10}}{0.014673846}\right](1.014673846)(1.06)^{10}} = 364.03$$

2. Jenna borrows 130,000 using a 10 year sinking fund loan. Under the terms of the loan, the interest on the loan of I will be paid annually for ten years using an interest rate of 6%. Additionally, at the end of each year, Jenna will make a deposit of D into the sinking fund. The deposit is determined such that the amount in the sinking fund at the end of ten years will be equal to the loan. The sinking fund will earn an annual effective interest rate of 5%.

Determine $D-I$.

Solution:

$$I = iL = (0.06)(130,000) = 7800$$

$$D = \frac{L}{s_{\overline{10}|0.05}} = \frac{130,000}{\left(\dfrac{(1.05)^{10} - 1}{0.05} \right)} = 10,335.59$$

$$D - I = 10,335.59 - 7800.00 = 2535.59$$

3. Spencer is repaying a loan with n level annual payments of 8726.18. The interest in the fifth payment is 4945.11. The interest in the 10th payment is 4014.28.

Determine n.

Solution:

$$\text{Prin}_5 = 8726.18 - 4945.11 = 3781.07$$

$$\text{Prin}_{10} = 8726.18 - 4014.28 = 4711.90$$

$$\text{Prin}_5(1.05)^5 = \text{Prin}_{10} ==> 3781.07(1+i)^5 = 4711.90 ==> i = \left(\frac{4711.90}{3781.07}\right)^{1/5} - 1 = 0.045$$

$$Qv^{n-k+1} = \text{Prin}_k$$

$$8726.18\left(\frac{1}{1.045}\right)^{n-5+1} = 3781.07$$

$$\left(\frac{1}{1.045}\right)^{n-4} = \frac{3781.07}{8726.18} ==> (n-4)\left[\ln\left(\frac{1}{1.045}\right)\right] = \ln\left(\frac{3781.09}{8726.18}\right)$$

$$n = \frac{\ln\left(\frac{3781.09}{8726.18}\right)}{\ln\left(\frac{1}{1.045}\right)} + 4 = 23$$