MATH 373
Quiz 6
Fall 2018

November 29, 2019

1. Liu Life Insurance Company has promised to pay Fang a payment of 500,000 at the end of 8 years.

   Liu Life Insurance Company wants to immunize the payment to Fang by investing the following two assets:

   i. Asset A is a zero coupon bond maturing for 10,000 at the end of 2 years.
   ii. Asset B is a zero coupon bond maturing for 10,000 at the end of 10 years.

   At an annual effective interest rate of 7%, calculate the number of each bonds that Liu should buy. (Assume that you can buy partial bonds.)

   Solution:

   Total price of asset A = $P_A = \left(\frac{500,000}{(1+i)^8}\right)\left(\frac{10-8}{10-2}\right) = 72,751.13807$

   price per A = $\frac{10,000}{(1.07)^2} = 8734.387283$

   Number of bond A = $\frac{72,751.13807}{8734.387283} = 8.329277797$

   Total price of Asset B = $P_B = \left(\frac{500,000}{(1.07)^8}\right)\left(\frac{6}{8}\right) = 218,253.4142$

   Price of Asset B = $\frac{10,000}{(1.07)^{10}} = 5083.492921$

   Number of Asset B = $\frac{218,253.4142}{5083.492921} = 42.93375$

2. Shikun is the recipient of an annuity that will pay the following:

   i. 100,000 today;
   ii. 200,000 at the end of two years; and
   iii. 300,000 at the end of four years.

Calculate the Macaulay Convexity of Shikun's payments at an interest rate of 4%.

Solution:

$$MacCon = \frac{\sum C_t \cdot t^2 \cdot v^t}{\sum C_t \cdot v^t} = \frac{(100,000)(0^2)(v^0) + (200,000)(2^2)v^2 + (300,000)(4^2)v^4}{100,000 + 200,000v^2 + 300,000v^4}$$

$$= \frac{4,842,705.087}{541,352.4999} = 8.945567053$$

3. A 20 year bond matures for its par value of 10,000. The bond pays annual coupons at a rate of 7%.

   Calculate the Modified duration of the bond using an interest rate of 5%.

   Solution:

$$Coupon = 10,000(0.07) = 700$$

N=20, PMT=700, FV=10,000, I/Y=5, CPT PV=12,492.44207 = Price of Bond

$$MacDur = \frac{\sum C_t \cdot t \cdot v^t}{\sum C_t \cdot v^t} = \frac{(700)(1)v^1 + (700)(2)v^2 + \ldots + (700)(20)v^{20} + (10,000)(20)v^{20}}{\text{Price of Bond}}$$

$$= \frac{700a_{\overline{20|}} + \frac{700}{0.05}(a_{\overline{20|}} - 20v^{20}) + (10,000)(20)v^{20}}{12,492.44207} = \frac{153,043.3334}{12,492.44207} = 12.25087397$$

$$ModDur = v(MacDur) = \frac{12.25087397}{1.05} = 11.6675$$