### 1.1 Introduction

**Chapter 1**

**The Growth of Money**

<table>
<thead>
<tr>
<th>1.15 Problems, Chapter 1</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.16 Introduction</td>
</tr>
<tr>
<td>1.14 Initial</td>
</tr>
<tr>
<td>1.13 Note for those who skipped Sections (1.11) and (1.12)</td>
</tr>
<tr>
<td>1.12 Force of Interest</td>
</tr>
<tr>
<td>1.11 A Briefly Introduction (Constant Force of Interest)</td>
</tr>
<tr>
<td>1.10 Nominal Rates of Interest and Discount</td>
</tr>
<tr>
<td>1.9 Compound Discount</td>
</tr>
<tr>
<td>1.8 Simple Discount</td>
</tr>
<tr>
<td>1.7 Discount Functions / The Time Value of Money</td>
</tr>
<tr>
<td>1.6 Interest in advance / The Effective Discount Rate</td>
</tr>
<tr>
<td>1.5 Compound Interest (The Usual Case)</td>
</tr>
<tr>
<td>1.4 Simple Interest / Linear Accumulation Functions</td>
</tr>
<tr>
<td>1.3 Accumulation and Amount Functions</td>
</tr>
<tr>
<td>1.2 What is Interest?</td>
</tr>
</tbody>
</table>

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**Example Calculations**

- **Example (a)**: With your calculator set to use the chain calculation method, consider the following sequence of keystrokes: In each case, describe what they accomplish if the calculation has just been entered.
- **Example (b)**: Calculate the future value of $100 invested at 5% interest compounded annually for 3 years.
- **Example (c)**: Determine the present value of $100 to be received in 3 years, discounted at 5%.
- **Example (d)**: Find the interest earned on $100 invested at 5% for 3 years.
- **Example (e)**: Calculate the effective interest rate for a loan with monthly payments.
Accumulation and amount functions

13.3 Accumulation and amount functions

Interest is paid on the basis of the balance of interest and the amount of interest. If you have more interest, the amount of interest will be more. The formula for calculating interest is:

\[ \text{Interest} = \text{Principal} \times \text{Rate} \times \text{Time} \]

The growth of money, interest, and capital is illustrated in this equation. The principal is the initial amount of money invested, the rate is the interest rate, and the time is the period for which the money is invested.

The amount function is used to calculate the future value of an investment. The formula for the amount function is:

\[ \text{Amount} = \text{Principal} \times (1 + \frac{\text{Rate}}{\text{Number of Compounding Periods}})^{\text{Number of Compounding Periods} \times \text{Time}} \]

The amount function takes into account the effect of compounding interest, which means that interest is earned on the interest that has already been earned. This results in a higher amount of money if the interest is compounded more frequently.

The growth of money is illustrated in this equation. The principal is the initial amount of money invested, the rate is the interest rate, the number of compounding periods is the number of times the interest is compounded, and the time is the period for which the money is invested.

One of the most widely accepted concepts for interest is the idea that money is a measure of the amount of money that is being earned. The amount of money that is being earned is the interest that is being paid on the principal. The rate of interest is the percentage of the principal that is being paid as interest. The time is the period for which the money is invested.

The growth of money, interest, and capital is illustrated in this equation. The principal is the initial amount of money invested, the rate is the interest rate, the number of compounding periods is the number of times the interest is compounded, and the time is the period for which the money is invested.

Section 1.2 What is interest?

Section 1.3 Accumulation and amount functions

Chapter 1 The growth of money

Accumulation and amount functions

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EXAMPLE 1.3.2

The associated accumulation and amount functions are step functions.

The growth of interest is proportional to the amount of money in the account at the end of each quarter. The interest rate of 6% per year means that the interest earned in each quarter is 1.5% of the amount of money in the account at the end of the previous quarter.

If interest is only paid at the end of each year, the graph of $A^1(t)$ is as follows:

Equations: $A^1(0) = 1$ and $A^1(t) = (1.06)^{t-1} A^1(t-1)$ for $t > 1$.

Graph of $A^1(t)$ is a line segment with slope 25%.

Solution

If interest is only paid at the end of each year, the amount function is $A^1(t)$ and the growth of money is $A^1(t)$.

Example 1.3.3

The growth of money $A^1(t)$ is a line segment.
Graph of \( a(n) = 5 \): For \( t = 3 \) the (i) \( u \) is 1. Thus, the function \( a(3) = (i) u \) is called the annual period and

\[
(i) u + 1 \cdot (1 - u) u = (i) u u \quad \text{and} \quad \frac{(1 - u) u}{(1 - u) u - (u) u} = \frac{(i) u}{(i) u - (u) u} = \frac{(i) u}{(i) u - (u) u} = (i) u
\]

We agree to write \( i \) for \( u - 1 \).

If \( n \) is a positive integer, the interval \( [n - 1, n] \) is called the \( n \)-th period and

\[
(i) n \neq (i) n
\]

This only happens if \( \tfrac{1}{n} \leq \tfrac{1}{n} \), for \( n \) is not equal to the reciprocal of \( 1 \). Of course, the interval \( [n - 1, n] \) cannot be the interval that begins with \( 1 \). (\( i \) is the period of interest for \( n \)) \( (i) n \) \( (i) n \) \( (i) n \) \( (i) n \)

It is convenient to note whenever \( (i) n \).

\[
\frac{(i) n}{(i) n - (u) n} = \frac{(i) n}{(i) n - (u) n} = \frac{(i) n}{(i) n - (u) n} = (i) n
\]

Note that whenever \( (i) n = (i) n \).

\[
\frac{(i) n}{(i) n - (u) n} = \frac{(i) n}{(i) n - (u) n} = \frac{(i) n}{(i) n - (u) n} = (i) n
\]

Effective interest rate for the interval \([i, i+1]\):

If \( i \leq 0 \), then \( \delta \leq 0 \).

\[
\delta \leq 0 \quad \text{for} \quad (i) \delta = \frac{(i) \delta}{(i) \delta - (u) \delta} = (i) \delta
\]

Solution

Graph of the associated accumulation function

\[
\delta \geq 0 \quad \text{for} \quad (i) \delta = \frac{(i) \delta}{(i) \delta - (u) \delta} = (i) \delta
\]

Example 1.3.4

The growth of money
Simple Interest

Ordinary simple interest is sometimes referred to as the "30/30/30/30 (1p - p) + (1m - m) + (1d - d) = p\]

(5.1.4)

You then compute the number of days using the formula:

\[\frac{[(1-u)+1]}{[(1-u)+1]-(u+n)} = \frac{1}{n}\]

When the growth of money is governed by simple interest, the interest is

Lower interest rates require Bop to pay more interest on their loans since the rate of interest is less.

Note that the annual effective interest rates are decreasing. Each year there is less

and therefore Bop must pay 0.8% of 8% = 0.64% of the loan for interest.

Interest rate on a loan is 1/2% of the loan on 9/19/99. When the loan matures on October 1, 1999, the simple

Interest formula is very similar to the loan on 10/9/99, the simple interest is

EXAMPLE 1.4.3

Exact Simple Interest: annual/quarterly/montly/daily

1993: and 1900 were not leap years.

Note to a leap year means 1985, 1996, and 2000 were leap years, while 1987 is a common year.

1987 is not divisible by 400. Hence 1985 is a leap year. The number of days from January 1 to December 31 is

28 + 31 + 31 + 30 + 31 + 30 + 31 + 31 + 30 + 31 + 30 + 31 = 365.

The leap year formula is:

\[\frac{[(1-u)+1]}{[(1-u)+1]-(u+n)} = \frac{1}{n}\]

EXAMPLE 1.4.2

Problem: Bop borrows $5,000 from July 1, 1999 at 8% simple interest.

1. How much does he pay by July 1, 1999?
2. How much does he pay by July 1, 2000?
3. How much does he pay by July 1, 2001?
4. How much does he pay by July 1, 2002?
5. What is the total amount Bop pays by July 1, 2002?
Chapter 1.5

Section 1.5

Compound Interest (The Usual Case)

15. Compound Interest is a valuable technique for interpreting facts about the non-

example (1.4.4) shows that the loan of Example (1.4.4) is made at 6% simple interest.

EXAMPLE 1.4.7

The banker's rate actual/360

The base of the loan is calculated as in Example (1.4.4) and we use 0.06.

The loan is made at 6% simple interest. What is the

Solution

According to the method of ordinary simple interest, the loan is made at 6% simple interest.

Problem

Suppose that the loan of Example (1.4.4) is made at 6% compound interest.

EXAMPLE 1.4.8

Ordinary Simple Interest 30/360

The growth of money

The notation used is (1.4.4) shows that the loan of Example (1.4.4) is made at 6% simple interest.

EXAMPLE 1.4.9

Date: 2000-09-07

Press: 2ND DATE

If you need to check whether this calculation is correct, simply

The due date of the loan is 2001-05-20.

The loan is made at 6% simple interest. What is the

Solution

According to the method of ordinary simple interest, the loan is made at 6% simple interest.

Problem

According to the method of ordinary simple interest, the loan is made at 6% simple interest.

EXAMPLE 1.4.10

The banker's rate actual/360

The base of the loan is calculated as in Example (1.4.4) and we use 0.06.

The loan is made at 6% simple interest. What is the

Solution

According to the method of ordinary simple interest, the loan is made at 6% simple interest.

Problem

According to the method of ordinary simple interest, the loan is made at 6% simple interest.
You can easily note that if you go enough to the right (large $t$), the compound interest is larger than the simple interest. The small graph on our right shows how the simple interest function is above the compound interest function. Your answer to our graphs shows how you can compute the future value of an account.

**Graph of $	ext{Compound Interest (usual case)}$**

The graph shows both kinds of accumulation with annual rates of 7%.

You might wonder how simple interest and compound interest compare. Here:

For $0$, $t = 1$: $f(t + 1) = (1 + t)$. 

For $a$, $b$, and $c$, the compound interest is $a$, and the simple interest is $b$. 

For accounts governed by the compound interest accumulation function at interest $f(1)$, money earns interest at the constant interest rate. As interest is paid, it is added to the account's principal, increasing the total amount in the account.

**Section 1.5**
Chapter 1.5

Section 1.5 Compound Interest (The usual case) 23

Problem: A bank offers a "promotional account" with compound interest at an annual effective interest rate of 2% on balances of at least $2,000. How much money does Fernando receive in this account?

Solution: The interest is compound, so $12,200.00 0.02 = $12,200.00 - $12,000.00 = $200.00.

Example 1.5.6

Theorem Bank account.

Example

Her balance is close to Fernando's, but is slightly higher.

Solution: Since the balance is close to Fernando's, we use the same formula for the interest rate. The balance is $12,300.00 in the account.

Problem: Find the difference from the function $f(x) = x^2 + 1$ at $x = 10$.

Solution: The difference is $f(10) - f(9) = (10^2 + 1) - (9^2 + 1) = 100 + 1 - 81 - 1 = 19$.

Problem: Find the greatest integer which does not exceed 1.

Solution: The function is defined by $f(x) = \lfloor x \rfloor$ for all $x$. The greatest integer in $\lfloor x \rfloor$ is the largest integer which does not exceed $x$.

Problem: Fernando's account balance is $12,000.00. How much money does Fernando receive in this account?

Solution: The interest is compound, so $12,000.00 0.02 = $240.00.

Example 1.5.4

Compound interest from deposit to withdrawal.

Problem: Fernando deposits $12,000.00 in an account at 2% effective interest rate. How much money does Fernando receive in this account?

Solution: Fernando's account balance is $12,000.00 (1.02)^n = $12,000.00 1.02^n = $12,240.00.

Theorem Bank account.

Example

Her balance is close to Fernando's, but is slightly higher.

Solution: Since the balance is close to Fernando's, we use the same formula for the interest rate. The balance is $12,300.00 in the account.

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Solution: The difference is $f(10) - f(9) = (10^2 + 1) - (9^2 + 1) = 100 + 1 - 81 - 1 = 19$.

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Example 1.5.4

Compound interest from deposit to withdrawal.

Problem: Fernando deposits $12,000.00 in an account at 2% effective interest rate. How much money does Fernando receive in this account?

Solution: Fernando's account balance is $12,000.00 (1.02)^n = $12,000.00 1.02^n = $12,240.00.
Section 1.6 Interest in advance / The effective discount rate

In other words, the discount rate for the complete amount of the loan is based on the effective interest rate, which is the amount of interest earned on the loan before the discount is applied.

\[
\text{Interest in advance = } P \times \frac{r}{100} \times t
\]

Where:
- \( P \) = principal amount
- \( r \) = annual interest rate
- \( t \) = time period

\[
\text{Discount} = P \times \frac{r}{100} \times t
\]

\[
\text{Effective discount rate} = \frac{\text{Discount}}{P} \times 100
\]

### Example 1.6.1

If \( P = 1000 \) and \( r = 7\% \), then the discount rate is:

\[
\text{Discount} = 1000 \times \frac{7}{100} = 70
\]

\[
\text{Effective discount rate} = \frac{70}{1000} \times 100 = 7\%
\]

### Example 1.6.2

If \( P = 1000 \) and \( r = 7\% \), then the discount rate is:

\[
\text{Discount} = 1000 \times \frac{7}{100} = 70
\]

\[
\text{Effective discount rate} = \frac{70}{1000} \times 100 = 7\%
\]

### Example 1.6.3

If \( P = 1000 \) and \( t = 1 \) year, then the discount rate is:

\[
\text{Discount} = 1000 \times \frac{7}{100} \times 1 = 70
\]

\[
\text{Effective discount rate} = \frac{70}{1000} \times 100 = 7\%
\]
\begin{align*}
\frac{u^p - 1}{u^p} & = u_i \\
1 & = (u^p - 1)(u + 1) \\
\text{where } u & = e^{rt} \\
\text{and } u_i & = e^{r_i t} \\
\text{Equation (11.6.1)}
\end{align*}

\begin{align*}
\frac{[e^{r_i t} - 1]}{[e^{r_i t}] - 1} & = [e^{r_i t}] \\
\text{Equation (11.6.1)}
\end{align*}

\begin{align*}
\frac{[e^{r_i t}] p - 1}{[e^{r_i t}] p} & = [e^{r_i t}] \\
\text{Equation (11.6.1)}
\end{align*}

\begin{align*}
\text{If we have a positive interest rate } r_i 	ext{ that the} \\
\frac{[e^{r_i t}] p - 1}{[e^{r_i t}] p} & = [e^{r_i t}] \\
\text{Equation (11.6.1)}
\end{align*}

\begin{align*}
\text{Now, let } [e^{r_i t}] p - [e^{r_i t}] & = 0 \\
\text{Equation (11.6.1)}
\end{align*}

\begin{align*}
\text{Interest is earned on all principal. If follows that the} \\
\text{interest rate is } \frac{[e^{r_i t}] p - 1}{[e^{r_i t}] p} = 1 \\
\text{Equation (11.6.1)}
\end{align*}

\begin{align*}
\text{Recall that } u = e^{rt} \text{ and that } u_i = e^{r_i t} \\
\text{Equation (11.6.1)}
\end{align*}

\begin{align*}
\text{IMPORTANT DEFINITION 11.6.2} \\
\text{Interest in advance / The effective discount rate}
\end{align*}

\begin{align*}
\text{Section 11.6} \\
\text{Interest in advance / The effective discount rate}
\end{align*}

\begin{align*}
\text{The growth of money is governed by the accumulation function.}
\end{align*}

\begin{align*}
\text{Example 11.6.5} \\
\text{Calculating Interest and discount rates: solution}
\end{align*}

\begin{align*}
\text{Compute this definition with the definition of } u \text{ given in Equation (11.3.7).}
\end{align*}

\begin{align*}
\text{Thus, we agree with the definition for } p \text{ for which} \\
\text{Recollect that } u = e^{rt} \text{ is a positive integer, the integral } u [1 - (1 - u)] = n \text{ is called the } \text{time.}
\end{align*}

\begin{align*}
\text{If, as is customary the case, } \frac{(1)iXV}{(1)iXV - (1)iXV} = \frac{[e^{r_i t}] p}{[e^{r_i t}] p} \\
\text{completes the interest rate } r_i \text{ as the } \text{accumulated amount of } [e^{r_i t}] p \text{ divided by the accumulated amount of } [e^{r_i t}] p \text{ and of the integral } [e^{r_i t}] p \text{ on } (1)XV \text{ the } \text{beginning of the interval.}
\end{align*}
\[
\frac{a}{i} = \frac{(i + 1)}{i} = \frac{(i)^1}{i} = (i)
\]

We define the discount factor

\[
(1 + i) = \frac{d}{i}
\]

Next, continue on the compound interest accumulation function of

\[
(1 + i)^n = \frac{(i)^n}{i} = (i)^n
\]

From now, we should invest \$7,200 now in order to have \$5,000 in 5 years.

The next example considers a more subtle question since you wish to invest at a

\[
\text{Problem: Suppose that the growth of money for the next five years is governed by the interest rate of} 8\%
\]

\[
\frac{(i)^5 + 1}{i} = (i)^5
\]

\[
\text{Solution: Suppose that the growth of money is governed by compound interest at an annual rate of} 8\%.
\]

\[
\frac{(i)^5 + 1}{i} = (i)^5
\]

\[
(1.08)^5 = 1.469
\]

\[
\text{and}
\]

\[
\text{The growth of money}
\]

\[
28
\]

Chapter 1
Another way of contrasting investments is by calculating their net present values.

Example 1.7.8: Involved in a comparison of investments using net present values.

\[
\text{Net present value} = \sum_{t=0}^{\infty} \frac{A_t}{(1+r)^t} \quad \text{where} \quad r = \text{interest rate}
\]

Suppose that the growth of money is viewed as being governed by an annuity.

\[
\text{Example 1.7.9:} \quad \text{Present value}\]

We define the present value of an annuity as the sum of the present values of each of the payments in the annuity. If we assume that each payment is equal, we can use the formula for the present value of an annuity.

\[
\text{Present value of annuity} = \frac{A_0}{r} \left(1 - \frac{1}{(1+r)^n}\right)
\]

where

- \(A_0\) is the initial payment
- \(r\) is the interest rate
- \(n\) is the number of payments

We can use this formula to calculate the present value of an annuity for different values of \(A_0\), \(r\), and \(n\).
At this point it is advisable to clear the worksheet by pushing $\boxed{CF} = 0$.

To open the cash Flow worksheet of the BA II Plus calculator push $\boxed{CF}$.

You should now notice that the display is cleared to zero.

The cash Flow section of the worksheet may also be calculated using the BA II Plus calculator and the associated NPV worksheet.

The NPV is a measure of the present value of a stream of future cash flows. It is calculated by discounting each cash flow back to the present using a discount rate. The NPV is positive if the present value of the future cash flows is greater than the initial investment, indicating that the investment is expected to be profitable. If the NPV is negative, the investment is expected to lose money.

Example 1.7.9

The growth of money 33
Section 17

Discount functions / The time value of money

35
1.8 Simple Discount

If possible, fill all the registers and press the "INS" key. If not, already filled, the display will indicate "M". The display may indicate "M" if any register is not full, or if your cash register is being used to store data.

The discount rate of the transaction will be entered as the negative of the discount rate. The discount rate may be changed at any time. To change the discount rate, press the "DISP" key and enter the new discount rate. The discount rate may be changed at any time. To change the discount rate, press the "DISP" key and enter the new discount rate.

1.9 Cash Flow Worksheet and NPV Subworksheet

This example is similar to the previous example. It assumes that you have a cash flow at time zero.

The cash flow at time zero is the present value of the investment. The cash flow at time zero is the present value of the investment. The cash flow at time zero is the present value of the investment.

In our next example, we will use the skills introduced in Example 1.11.14 to calculate the net present value of the investment.

Now, press the "DISP" key and enter the net present value.

The net present value is the discounted cash flow at the beginning of the project. The net present value is the discounted cash flow at the beginning of the project. The net present value is the discounted cash flow at the beginning of the project.

Since the cash flow is discounted, the result is expressed as a fraction. To express the result as a fraction, divide the numerator and denominator by the common denominator. To express the result as a fraction, divide the numerator and denominator by the common denominator.

Once again, you may choose to express the result as a fraction. To express the result as a fraction, divide the numerator and denominator by the common denominator.

The result will be displayed as a fraction. To express the result as a fraction, divide the numerator and denominator by the common denominator.

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The result will be displayed as a fraction. To express the result as a fraction, divide the numerator and denominator by the common denominator.
For if only makes sense to talk about simple discount on the interval $[0, t]$. Then $a(t)$ is an increasing function that is asymptotic to the line $x = t$. Therefore, $a(t) = \frac{t p - 1}{1}$.

Hence, the simple discount accumulation function is

$$I(t) = \frac{t p - 1}{1} = (t) a$$

So the simple discount accumulation function is

$$I = \frac{t p - 1}{1} = (t) a$$

and

$$I = \frac{t p - 1}{1} = \frac{t + t f}{1} = \frac{t + t f}{1}$$

Observing the graph of $I(t)$ in Figure 1.9, we see that the function is increasing. Since $a(t)$ is an increasing function that is asymptotic to the line $x = t$, we say that we have simple discount. In fact, $I(t) = (t) a$

Graph of $a(t)$

![Graph of a(t)](image)

The graph of money growth of $s$ for $t = (t s + 1)$.
Example 1.9.9 \[ \text{is useful here.} \]

Ineffective discount rate:

This is because the effective rate of an amount, or in this case the interest rate, is not affected by the amount borrowed. In other words, the effective rate remains the same regardless of the amount borrowed.

Example 1.9.8

To obtain \( q \) (there NOT as a percent),

\[ \frac{j + 1}{r} - 1 = p \]

Once \( j \) and \( r \) are known, \( q \) is obtained using equation (1.9.12).

\[ \frac{j + 1}{r} - 1 = p \]

Interest due on \$45870.27 for 4.4\% annual effective rate:

\[ - \frac{p}{r} \]

Solution 2

Square Root

\[ \frac{x}{r} = \sqrt{1 + \frac{p}{r}} \]

Solution 1

Use formula (1.9.12) to obtain \( q \). On the BA Plus calculator, push

\[ \frac{x}{r} = \sqrt{1 + \frac{p}{r}} \]

Annual effective interest rate, or an interest rate of \( p \), is known.

Example 1.9.3

Finding an effective interest rate equivalent to a

\[ \frac{x}{r} = \sqrt{1 + \frac{p}{r}} \]

Solution 1

Use formula (1.9.12) to obtain \( q \). On the BA Plus calculator, push

\[ \frac{x}{r} = \sqrt{1 + \frac{p}{r}} \]

Annual effective interest rate is 4.36826%.

Example 1.9.2

Given annual effective rate is 4.36826%.

To find equivalent rate of interest:

\[ \frac{j + 1}{r} - 1 = p \]

Example 1.9.1

Use formula (1.9.12) to obtain \( q \). On the BA Plus calculator, push

\[ \frac{x}{r} = \sqrt{1 + \frac{p}{r}} \]

Annual effective interest rate is 4.36826%.

Example 1.9.0

Finding an effective interest rate equivalent to a
Suppose that we have an investment governed by compound interest. This means that if you invest $X$ now at an annual interest rate of $r$%, the amount of money you will have $t$ years later will be $X(1 + r)^t$.

### Example 1

**FIGURE 1.9.12**

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>$r$</th>
<th>$X(1 + r)^t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>$r$</td>
<td>$X$</td>
</tr>
</tbody>
</table>

The amount you receive at time zero is $0 = X$ when $t = 0$. Thus, if you receive $X$ from $t = 1$, you receive $X(1 + r)$ from $t = 2$, and so on. The amount of money you will have accumulated after $t$ years is $X(1 + r)^t$.

### Section 1.10

Nominal Rates of Interest and Discount

The effective interest rate per year on an investment governed by compound interest is given by the formula $1 + r = (1 + i)^t$, where $i$ is the annual interest rate and $t$ is the number of years.

### Problem

The amount of money you receive from an investment made at time zero after $t$ years is $X(1 + r)^t$. If you receive $X$ now, how much money will you receive at time $t$?

### Solution

The amount of money you receive at time $t$ is $X(1 + r)^t$.

### Figure 1.9.13

<table>
<thead>
<tr>
<th>$t$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>$r$</th>
<th>$X(1 + r)^t$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>2</td>
<td>$r$</td>
<td>$X$</td>
</tr>
</tbody>
</table>

1. **Example 1.9.12**

- If you invest $100 at 5%, how much money will you have after 2 years?
- If you invest $200 at 7%, how much money will you have after 3 years?

2. **Problem 1.9.13**

- If you receive $500 now, how much money will you receive after 5 years at 4% interest?
- If you receive $800 now, how much money will you receive after 10 years at 3% interest?
The following important statement just observed when the rate for such an interval is $\left(1 - \frac{w}{(w-1)} \right) = \frac{w}{(w-1)}$, and the formula for annual effective yield is $\left(1 - \frac{w}{(w-1)} \right)^{1/n} = \frac{w}{(w-1)}$. Therefore, the annual effective yield is $\left(1 - \frac{w}{(w-1)} \right)^{1/n}$. Therefore, the annual effective yield is $\left(1 - \frac{w}{(w-1)} \right)^{1/n}$.
\[ P < (w)P < (w)! < 1 \]

\[ \text{IMPORTANT FACT: If } 0 < (w)! < 1 \text{ and } w > 0, \text{ then } 0 < (w)P < (w)! < 1 \text{ for all } t \geq 0. \]

Thus, from 1.1.3, we need to ensure that the discount rate is less than 1 to ensure that the present value is less than the future value.

\[ \frac{w}{(w)!} > (w)! \quad \text{and} \quad \frac{w}{(w)!} - 1 = (w)P \]

\[ \frac{w}{(w)!} + 1 = (w)! \quad \text{and} \quad \frac{w}{(w)!} - 1 = (w)P \]

Therefore, we observe that by increasing the discount rate, the present value decreases, which is consistent with the economic intuition that borrowing money costs more than lending it.

\[ \frac{w}{(w)!} > (w)! \quad \text{and} \quad \frac{w}{(w)!} - 1 = (w)P \]

\[ \frac{w}{(w)!} + 1 = (w)! \quad \text{and} \quad \frac{w}{(w)!} - 1 = (w)P \]

From which, one easily obtains:

\[ \frac{w}{(w)!} = \frac{w}{(w)!} \quad \text{and} \quad \frac{w}{(w)!} - 1 = (w)P \]

This is equivalent to:

\[ 1 = (\frac{w}{(w)!} + 1) (\frac{w}{(w)!} - 1) \]

Just as we derived formula (1.6)'s, we can show that:

\[ \frac{w}{(w)!} (P - 1) = w = (w)P \]

Therefore:

\[ \frac{w}{(w)!} (P - 1) = w = (w)P \]

This problem demonstrates the relationship between a nominal interest rate and a discount rate.

**Example 1.10.15**

Illustrate this formula with different compounded rates and a discount rate.

This equation is useful for converting between a nominal interest rate and a discount rate.

\[ \frac{d}{d - (d)P} = 1 - (P - 1) = 1 + \frac{w}{(w)!} + 1 \]

(1.10.1)

This holds for any integers, and also holds where both \( d \) and \( w \) are integers, since Equations 1.1.3 (1.10.1) and (1.10.1)

\[ \frac{w}{(w)!} + 1 = (w)! \quad \text{and} \quad \frac{w}{(w)!} - 1 = (w)P \]

\[ \frac{w}{(w)!} + 1 = (w)! \quad \text{and} \quad \frac{w}{(w)!} - 1 = (w)P \]

From (1.10.1) we also derive:

Section 1.10: Nominal Rates of Interest and Discount

Chapter 1: The Growth of Money
Section 1.10
Nominal rate of interest and discount

49

Nominal rate of interest and discount

Equation 1.10-20

\[ \left( \frac{w}{(aw)^{P-1}} + 1 \right) - 1 = p \]

If you have an effective interest rate and desire an equivalent nominal rate,

\[ \text{EFF} = \text{EF} \]

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\[ \text{6} \]

\[ \text{CONV} \]

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\[ \text{CF} \]

\[ \text{CONV} \]

\[ \text{6} \]

\[ \text{CONV} \]

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**EXAMPLE 1.10.23**

1.11 A FRIENLY COMPARISON (Constant Force of Interest)

Suppose the interest rate is 3.2% per year. Then the equivalent interest rate at which the
numerical value of \( \frac{1}{1.032} \) is the same as the numerical value of \( \frac{1.032}{1} \) is 3.2%.

(a) \( \frac{1}{1.032} \)

Solution:

1. \( \frac{1}{1.032} \) is the equivalent interest rate.
2. \( \frac{1.032}{1.032} \) is the numerical value of \( \frac{1}{1.032} \) as a percent.

Problem: Use the BA II Plus calculator to find an annual effective discount rate.

**EXAMPLE 1.10.24**

1.11 A FRIENLY COMPARISON (Constant Force of Interest)

Section 1.11: A Friendly Comparison (Constant Force of Interest) 3.2%
Problem: Money at National Bank grows at 5% compounded continuously.

Example 1.1.7

Solution: We are given that \( i = \frac{9}{12} \) and \( P = 1200 \). Hence, the amount of money after 5 years is

\[
A = P \cdot e^{rt} = 1200 \cdot e^{\frac{9}{12} \cdot 5} = 1200 \cdot e^{0.75} = 2024.87
\]

Therefore, the amount of money after 5 years is approximately $2024.87.

In conclusion, the formula for compound interest, \( A = P \cdot e^{rt} \), is valid and can be used to calculate the amount of money after a certain period of time.

Section 1.1.8

A Friendly Competition (Constant Force of Interest)

Problem: Suppose the force of interest is

\[
\phi(t) = e^{-0.5t}
\]

Find the amount of money accumulated after 3 years.

Solution: We are given that \( \phi(t) = e^{-0.5t} \) and \( t = 3 \). Hence, the amount of money after 3 years is

\[
A = P \cdot e^{\int_{0}^{t} \phi(u) \, du} = P \cdot e^{\int_{0}^{3} e^{-0.5u} \, du}
\]

\[
= P \cdot e^{[-e^{-0.5u}]_{0}^{3}} = P \cdot e^{-0.5 \cdot 3} = P \cdot e^{-1.5}
\]

Therefore, the amount of money after 3 years is

\[
A = P \cdot e^{-1.5}
\]

Problem: Suppose the force of interest is

\[
\phi(t) = e^{-t}
\]

Find the amount of money accumulated after 5 years.

Solution: We are given that \( \phi(t) = e^{-t} \) and \( t = 5 \). Hence, the amount of money after 5 years is

\[
A = P \cdot e^{\int_{0}^{t} \phi(u) \, du} = P \cdot e^{\int_{0}^{5} e^{-u} \, du}
\]

\[
= P \cdot e^{[-e^{-u}]_{0}^{5}} = P \cdot e^{-5}
\]

Therefore, the amount of money after 5 years is

\[
A = P \cdot e^{-5}
\]

Problem: Suppose the force of interest is

\[
\phi(t) = e^{-0.25t}
\]

Find the amount of money accumulated after 10 years.

Solution: We are given that \( \phi(t) = e^{-0.25t} \) and \( t = 10 \). Hence, the amount of money after 10 years is

\[
A = P \cdot e^{\int_{0}^{t} \phi(u) \, du} = P \cdot e^{\int_{0}^{10} e^{-0.25u} \, du}
\]

\[
= P \cdot e^{[-e^{-0.25u}]_{0}^{10}} = P \cdot e^{-1}
\]

Therefore, the amount of money after 10 years is

\[
A = P \cdot e^{-1}
\]

Problem: Suppose the force of interest is

\[
\phi(t) = e^{-0.5t}
\]

Find the amount of money accumulated after 7 years.

Solution: We are given that \( \phi(t) = e^{-0.5t} \) and \( t = 7 \). Hence, the amount of money after 7 years is

\[
A = P \cdot e^{\int_{0}^{t} \phi(u) \, du} = P \cdot e^{\int_{0}^{7} e^{-0.5u} \, du}
\]

\[
= P \cdot e^{[-e^{-0.5u}]_{0}^{7}} = P \cdot e^{-3.5}
\]

Therefore, the amount of money after 7 years is

\[
A = P \cdot e^{-3.5}
\]
The growth of money

(i) gives a second formula with which to calculate the force of interest $q$ from

$$\frac{(i)^{n+1}q^n}{p} = q$$

(1.12)

The definition (1.12) of the force of interest $q$ tells us how to obtain $q$ from

Therefore, $q = (i)^{n+1}q^n = (i)^n q = q$

(1.13)

Observing that the ratio $\frac{(i)^n q^n}{p}$ is equal to the derivative

$$\frac{d}{dx} (i)^n q^n = (i)^{n+1} q^n$$

(1.14)

the accumulation function $q(x)$ is a function of $x$. This is because the accumulation function $q(x)$ is defined as the force of interest at time $t$.

Now let's return to the problem of finding the force of interest $q$.

EXAMPLE 1.12.4 For compound interest

$$\frac{(i)^{n+1}q^n}{p} = q$$

(1.15)

The accumulation function $q(x)$ is a function of $x$. Therefore, $q = (i)^{n+1}q^n = (i)^n q = q$

(1.16)

The problem: Suppose that you have simple discount $d$ at rate $p$. Find the force of interest.

EXAMPLE 1.12.3 For simple discount

$$\frac{(i)^{n+1}q^n}{p} = q$$

(1.17)

The accumulation function $q(x)$ is a function of $x$. Therefore, $q = (i)^{n+1}q^n = (i)^n q = q$

(1.18)

The problem: Suppose that you have simple interest $i$ at rate $p$. Find the force of interest.

EXAMPLE 1.12.2 For simple interest

$$\frac{(i)^{n+1}q^n}{p} = q$$

(1.19)

The accumulation function $q(x)$ is a function of $x$. Therefore, $q = (i)^{n+1}q^n = (i)^n q = q$

(1.20)

The problem: Suppose that you have simple interest $i$ at rate $p$. Find the force of interest.

EXAMPLE 1.12.1 Force of Interest

$$\frac{(i)^{n+1}q^n}{p} = q$$

(1.21)

The definition (1.12) of the force of interest $q$ tells us how to obtain $q$ from

Therefore, $q = (i)^{n+1}q^n = (i)^n q = q$

(1.22)

The problem: Suppose that you have simple discount $d$ at rate $p$. Find the force of interest.

EXAMPLE 1.12.3 For simple discount

$$\frac{(i)^{n+1}q^n}{p} = q$$

(1.23)

The accumulation function $q(x)$ is a function of $x$. Therefore, $q = (i)^{n+1}q^n = (i)^n q = q$

(1.24)

The problem: Suppose that you have simple interest $i$ at rate $p$. Find the force of interest.

EXAMPLE 1.12.2 For simple interest

$$\frac{(i)^{n+1}q^n}{p} = q$$

(1.25)

The accumulation function $q(x)$ is a function of $x$. Therefore, $q = (i)^{n+1}q^n = (i)^n q = q$

(1.26)

The problem: Suppose that you have simple interest $i$ at rate $p$. Find the force of interest.

EXAMPLE 1.12.1 Force of Interest

$$\frac{(i)^{n+1}q^n}{p} = q$$

(1.27)
Example 1.13.1

The concept of interest is called the force of interest.

\( \delta = \frac{\ln(1 + r)}{t} = (t + 1) \ln(1 + r) = \frac{\ln(1 + r)}{t} \)

where \( \delta \) is the rate of interest and \( r \) is the nominal rate of interest.

The force of interest is related to the equivalent continuously compounded force of interest.

\[ \delta \] is the growth rate of the force of interest.

The growth of money is accelerated when the equivalent continuously compounded force of interest is approximated to in an annual effective rate of interest.

More generally, the equivalent continuously compounded force of interest is approximated to in an annual effective rate of interest.

Finally, the equivalent continuously compounded force of interest is approximated to in an annual effective rate of interest.

Therefore, the equivalent continuously compounded force of interest is approximated to in an annual effective rate of interest.

We next learn how the accumulation function \( a(t) \) may be found if the force of interest is known.

\[ a(t) = \frac{\ln(1 + r)}{t} \]

According to the formula, \( a(t) \) is a function of \( t \) and \( r \).

Example 1.13.2

Finding \( a(t) \) from \( \delta(t) \)

\[ \delta(t) = \frac{\ln(1 + r)}{t} \]

Therefore, the accumulation function is computed as:

\[ a(t) = \frac{\ln(1 + r)}{t} \]

The growth of money is:

\[ a(t) = \frac{\ln(1 + r)}{t} \]
Section 1.14 Inflation

Inflation is the rate at which a country's price level increases over time. It is a measure of the change in the average price of a basket of goods and services over time. Inflation erodes the purchasing power of money, meaning that a given amount of money will buy fewer goods and services over time.

1.14.1 Inflation

The growth rate of money is a key factor in determining inflation. If the growth rate of money is greater than the growth rate of output, inflation will occur. Conversely, if the growth rate of money is less than the growth rate of output, deflation may occur. The relationship between the growth rate of money and inflation can be expressed as:

\[ \frac{\Delta M}{\Delta Y} = \frac{\Delta P}{\Delta T} \]

where \( \Delta M \) is the change in the money supply, \( \Delta Y \) is the change in real GDP, \( \Delta P \) is the change in the price level, and \( \Delta T \) is the change in time.

1.14.2 The Growth of Money

The growth rate of money (\( \Delta M \)) is influenced by various factors, including monetary policy set by central banks. Central banks can influence the growth rate of money by adjusting the money supply. For example, if a central bank increases the money supply, the growth rate of money will increase, which can lead to inflation. Conversely, if a central bank decreases the money supply, the growth rate of money will decrease, which can lead to deflation.

Example 1.14.6 Anticipated Inflation

If you expect the inflation rate to be 4%, you would require a 4% increase in the nominal interest rate to achieve a real interest rate of 0%. This is known as anticipated inflation. If you do not expect inflation, you would require a lower nominal interest rate to achieve a real interest rate of 0%.

Example 1.14.7 Unanticipated Inflation

If you unexpectedly experience 5% inflation, you would require a 5% increase in the nominal interest rate to achieve a real interest rate of 0%. This is known as unanticipated inflation. If you anticipate the inflation rate, you would require a lower nominal interest rate to achieve a real interest rate of 0%.

1.14.3 The Growth Rate of Money

The growth rate of money (\( \Delta M \)) is determined by the money supply (\( M \)), which is influenced by monetary policy set by central banks. The growth rate of money is related to the growth rate of real GDP (\( \Delta Y \)) and the growth rate of the price level (\( \Delta P \)). If the growth rate of money is greater than the growth rate of real GDP, inflation will occur. Conversely, if the growth rate of money is less than the growth rate of real GDP, deflation may occur.

\[ \frac{\Delta M}{\Delta Y} = \frac{\Delta P}{\Delta T} \]

1.14.4 The Interest Rate on Investment

The interest rate on investment (\( i \)) is influenced by the real interest rate (\( r \)) and the price level (\( P \)). The real interest rate is the nominal interest rate adjusted for inflation. If the price level increases, the real interest rate will decrease, which can reduce investment.

\[ i = r + \pi \]

where \( \pi \) is the inflation rate.

1.14.5 The Fisher Equation

The Fisher equation relates the nominal interest rate (\( i \)) to the real interest rate (\( r \)) and the inflation rate (\( \pi \)). The equation is expressed as:

\[ i = r + \pi \]

This equation states that the nominal interest rate is equal to the real interest rate plus the inflation rate.
In this book, except when explicitly stated to the contrary, we will denote
that a variable or an equation is to be evaluated at a point, e.g.,
\[ f(a) = \left. f(x) \right|_{x=a} \]

1.9 Exponential growth

The growth of money

Example 1.9.7 Illustrating the importance of the denominator of
\[ e^{0.04t} \]

We note the answer to Question (c) of Example 1.4.7 was found using

\[ 100 \approx \frac{1053}{45} \approx 0.0442 = 4.42\% \]

Section 1.4.7

1.15 Problems, Chapter 1

Problems, Chapter 1
1.3 Accumulation and moment functions

Section 1.1.5

Problems, Chapter 1

63

4. If you are considering your money will stay on deposit for a total of 12 years, how much money will you have?
5. If you wish to withdraw your entire account balance at the end of 5 years, how much money will you have?
6. If you wish to withdraw your entire account balance at the end of 10 years, how much money will you have?
7. If your money is in a savings account earning simple interest, how much money will you have if the account is closed at the end of 5 years?
8. If you wish to withdraw your entire account balance at the end of 5 years, how much money will you have?
9. If your money is in a savings account earning simple interest, how much money will you have if the account is closed at the end of 10 years?
10. If you wish to withdraw your entire account balance at the end of 10 years, how much money will you have?

1.3.1 Simple Interest

Suppose you deposit $2,000 in a bank at an annual simple interest rate of 5%. After 1 year, how much interest will you earn?
1. If you deposit $2,000 in a bank at an annual simple interest rate of 5%, how much interest will you earn after 2 years?
2. If you deposit $2,000 in a bank at an annual simple interest rate of 5%, how much interest will you earn after 3 years?
3. If you deposit $2,000 in a bank at an annual simple interest rate of 5%, how much interest will you earn after 4 years?
4. If you deposit $2,000 in a bank at an annual simple interest rate of 5%, how much interest will you earn after 5 years?
5. If you deposit $2,000 in a bank at an annual simple interest rate of 5%, how much interest will you earn after 6 years?
6. If you deposit $2,000 in a bank at an annual simple interest rate of 5%, how much interest will you earn after 7 years?
7. If you deposit $2,000 in a bank at an annual simple interest rate of 5%, how much interest will you earn after 8 years?
8. If you deposit $2,000 in a bank at an annual simple interest rate of 5%, how much interest will you earn after 9 years?
9. If you deposit $2,000 in a bank at an annual simple interest rate of 5%, how much interest will you earn after 10 years?
10. If you deposit $2,000 in a bank at an annual simple interest rate of 5%, how much interest will you earn after 11 years?
11. If you deposit $2,000 in a bank at an annual simple interest rate of 5%, how much interest will you earn after 12 years?

1.3.2 Compound Interest

Suppose you deposit $2,000 in a bank at an annual compound interest rate of 5%. After 1 year, how much interest will you earn?
1. If you deposit $2,000 in a bank at an annual compound interest rate of 5%, how much interest will you earn after 2 years?
2. If you deposit $2,000 in a bank at an annual compound interest rate of 5%, how much interest will you earn after 3 years?
3. If you deposit $2,000 in a bank at an annual compound interest rate of 5%, how much interest will you earn after 4 years?
4. If you deposit $2,000 in a bank at an annual compound interest rate of 5%, how much interest will you earn after 5 years?
5. If you deposit $2,000 in a bank at an annual compound interest rate of 5%, how much interest will you earn after 6 years?
6. If you deposit $2,000 in a bank at an annual compound interest rate of 5%, how much interest will you earn after 7 years?
7. If you deposit $2,000 in a bank at an annual compound interest rate of 5%, how much interest will you earn after 8 years?
8. If you deposit $2,000 in a bank at an annual compound interest rate of 5%, how much interest will you earn after 9 years?
9. If you deposit $2,000 in a bank at an annual compound interest rate of 5%, how much interest will you earn after 10 years?
10. If you deposit $2,000 in a bank at an annual compound interest rate of 5%, how much interest will you earn after 11 years?
11. If you deposit $2,000 in a bank at an annual compound interest rate of 5%, how much interest will you earn after 12 years?
53.6% Find X. 33S% Find X. 39% Find X. 46% Find X.
(1) A discount bond is an investment in which the buyer agrees to pay the face value of the bond to the seller at a future date. The buyer will receive interest payments at specified intervals until the bond matures. The purchase price of a bond is equal to the present value of the future cash flows, discounted at the market interest rate.
(2) To calculate the present value of a bond, you need to know the face value of the bond, the interest rate, and the time to maturity. The formula for the present value of a bond is:

\[ PV = \frac{F}{(1 + r)^n} \]

where \( PV \) is the present value, \( F \) is the face value of the bond, \( r \) is the market interest rate, and \( n \) is the number of years until maturity.

(3) The amount of interest earned on a bond depends on the coupon rate and the time until maturity. The coupon rate is the annual interest payment divided by the face value of the bond. The time until maturity is the number of years until the bond matures. The formula for the interest earned on a bond is:

\[ I = \frac{F \times r}{P} \]

where \( I \) is the interest earned, \( F \) is the face value of the bond, \( r \) is the coupon rate, and \( P \) is the purchase price of the bond.

(4) To calculate the purchase price of a bond, you need to know the face value of the bond, the interest rate, and the time to maturity. The formula for the purchase price of a bond is:

\[ P = \frac{F}{(1 + r)^n} \]

where \( P \) is the purchase price of the bond, \( F \) is the face value of the bond, \( r \) is the market interest rate, and \( n \) is the number of years until maturity.

(5) The effective annual rate (EAR) is the annual interest rate that accounts for the effect of compounding interest over multiple periods. The formula for the EAR is:

\[ EAR = (1 + \frac{r}{n})^n - 1 \]

where \( r \) is the nominal interest rate and \( n \) is the number of compounding periods per year.

(6) The time value of money is the concept that money available at an earlier date is more valuable than the same amount of money available at a later date. This is because money can be invested and earn interest over time. The formula for the future value of a single payment is:

\[ FV = PV \times (1 + r)^n \]

where \( FV \) is the future value, \( PV \) is the present value, \( r \) is the interest rate, and \( n \) is the number of years.

(7) The present value of a series of payments is the sum of the present values of each individual payment. The formula for the present value of a series of payments is:

\[ PV = \sum_{i=1}^{n} \frac{C_i}{(1 + r)^i} \]

where \( PV \) is the present value, \( C_i \) is the cash flow in period \( i \), \( r \) is the interest rate, and \( n \) is the number of periods.

(8) The present value of a perpetuity is the sum of the present values of an infinite series of payments. The formula for the present value of a perpetuity is:

\[ PV = \frac{C}{r} \]

where \( PV \) is the present value, \( C \) is the annual cash flow, and \( r \) is the interest rate.

(9) The present value of a growing perpetuity is the sum of the present values of an infinite series of payments that grow at a constant rate. The formula for the present value of a growing perpetuity is:

\[ PV = \frac{C_0}{r - g} \]

where \( PV \) is the present value, \( C_0 \) is the initial cash flow, \( r \) is the interest rate, and \( g \) is the growth rate.

(10) The effective annual rate (EAR) is the annual interest rate that accounts for the effect of compounding interest over multiple periods. The formula for the EAR is:

\[ EAR = (1 + \frac{r}{n})^n - 1 \]

where \( r \) is the nominal interest rate and \( n \) is the number of compounding periods per year.

(11) The time value of money is the concept that money available at an earlier date is more valuable than the same amount of money available at a later date. This is because money can be invested and earn interest over time. The formula for the future value of a single payment is:

\[ FV = PV \times (1 + r)^n \]

where \( FV \) is the future value, \( PV \) is the present value, \( r \) is the interest rate, and \( n \) is the number of years.

(12) The present value of a series of payments is the sum of the present values of each individual payment. The formula for the present value of a series of payments is:

\[ PV = \sum_{i=1}^{n} \frac{C_i}{(1 + r)^i} \]

where \( PV \) is the present value, \( C_i \) is the cash flow in period \( i \), \( r \) is the interest rate, and \( n \) is the number of periods.

(13) The present value of a growing perpetuity is the sum of the present values of an infinite series of payments that grow at a constant rate. The formula for the present value of a growing perpetuity is:

\[ PV = \frac{C_0}{r - g} \]

where \( PV \) is the present value, \( C_0 \) is the initial cash flow, \( r \) is the interest rate, and \( g \) is the growth rate.
You have a choice of depositing your money in an account A in which there is an annual effective interest rate of 7.5% or in an account B which has an annual effective interest rate of 8%. Find the equivalent rate.

(1) Suppose we have compounded interest and discount.

\[ \text{Equivalent rate of } 8\% \text{ in an account with } (\frac{p}{p}) \text{ P}\]

\[ \text{Equivalent rate of } 8\% \text{ in an account with } (\frac{p}{p}) \text{ P}\]

(2) \( \text{Suppose we have simple interest and discount.} \)

\[ \text{Equivalent rate of } 8\% \text{ in an account with } (\frac{p}{p}) \text{ P}\]

(3) \( \text{Simple interest rate in an account with } (\frac{p}{p}) \text{ P}\)

(4) \( \text{The annual effective rate in an account with } (\frac{p}{p}) \text{ P}\)

(5) \( \text{The annual effective rate in an account with } (\frac{p}{p}) \text{ P}\)

(6) \( \text{The annual effective rate in an account with } (\frac{p}{p}) \text{ P}\)

Section 1.13 Problems, Chapter 1

Chapter 1 The growth of money

Page 67
The growth of money

Section 1.15

Problems' Chapter 1
Chapter 1: Review Problems

1. Assume an investment is governed by an accumulation function
   \[ A(t) = A_0 e^{rt} \]
   Where \( A_0 \) is the initial amount, \( r \) is the annual interest rate, and \( t \) is the time in years.

2. Suppose that an investment is governed by an accumulation function
   \[ A(t) = \frac{A_0}{1 - r} \]
   Where \( A_0 \) is the initial amount, \( r \) is the annual interest rate, and \( t \) is the time in years.

3. Suppose that an investment is governed by an accumulation function
   \[ A(t) = A_0 (1 + rt)^t \]
   Where \( A_0 \) is the initial amount, \( r \) is the annual interest rate, and \( t \) is the time in years.

4. Suppose that an investment is governed by an accumulation function
   \[ A(t) = \frac{A_0}{1 - r} \]
   Where \( A_0 \) is the initial amount, \( r \) is the annual interest rate, and \( t \) is the time in years.

5. Money is invested in a savings account with a nominal interest rate of 2%.
   \[ A(t) = A(0) e^{rt} \]
   Where \( A(0) \) is the initial amount, \( r \) is the annual interest rate, and \( t \) is the time in years.

6. Suppose that an investment is governed by an accumulation function
   \[ A(t) = A_0 (1 + rt)^t \]
   Where \( A_0 \) is the initial amount, \( r \) is the annual interest rate, and \( t \) is the time in years.

7. Suppose that an investment is governed by an accumulation function
   \[ A(t) = \frac{A_0}{1 - r} \]
   Where \( A_0 \) is the initial amount, \( r \) is the annual interest rate, and \( t \) is the time in years.

8. Suppose that an investment is governed by an accumulation function
   \[ A(t) = A_0 e^{rt} \]
   Where \( A_0 \) is the initial amount, \( r \) is the annual interest rate, and \( t \) is the time in years.

9. Suppose that an investment is governed by an accumulation function
   \[ A(t) = \frac{A_0}{1 - r} \]
   Where \( A_0 \) is the initial amount, \( r \) is the annual interest rate, and \( t \) is the time in years.

10. Suppose that an investment is governed by an accumulation function
    \[ A(t) = A_0 e^{rt} \]
    Where \( A_0 \) is the initial amount, \( r \) is the annual interest rate, and \( t \) is the time in years.

11. Suppose that an investment is governed by an accumulation function
    \[ A(t) = \frac{A_0}{1 - r} \]
    Where \( A_0 \) is the initial amount, \( r \) is the annual interest rate, and \( t \) is the time in years.

12. Suppose that an investment is governed by an accumulation function
    \[ A(t) = A_0 e^{rt} \]
    Where \( A_0 \) is the initial amount, \( r \) is the annual interest rate, and \( t \) is the time in years.

13. Suppose that an investment is governed by an accumulation function
    \[ A(t) = \frac{A_0}{1 - r} \]
    Where \( A_0 \) is the initial amount, \( r \) is the annual interest rate, and \( t \) is the time in years.

14. Suppose that an investment is governed by an accumulation function
    \[ A(t) = A_0 e^{rt} \]
    Where \( A_0 \) is the initial amount, \( r \) is the annual interest rate, and \( t \) is the time in years.

15. Suppose that an investment is governed by an accumulation function
    \[ A(t) = \frac{A_0}{1 - r} \]
    Where \( A_0 \) is the initial amount, \( r \) is the annual interest rate, and \( t \) is the time in years.

16. Suppose that an investment is governed by an accumulation function
    \[ A(t) = A_0 e^{rt} \]
    Where \( A_0 \) is the initial amount, \( r \) is the annual interest rate, and \( t \) is the time in years.

17. Suppose that an investment is governed by an accumulation function
    \[ A(t) = \frac{A_0}{1 - r} \]
    Where \( A_0 \) is the initial amount, \( r \) is the annual interest rate, and \( t \) is the time in years.

18. Suppose that an investment is governed by an accumulation function
    \[ A(t) = A_0 e^{rt} \]
    Where \( A_0 \) is the initial amount, \( r \) is the annual interest rate, and \( t \) is the time in years.

19. Suppose that an investment is governed by an accumulation function
    \[ A(t) = \frac{A_0}{1 - r} \]
    Where \( A_0 \) is the initial amount, \( r \) is the annual interest rate, and \( t \) is the time in years.

20. Suppose that an investment is governed by an accumulation function
    \[ A(t) = A_0 e^{rt} \]
    Where \( A_0 \) is the initial amount, \( r \) is the annual interest rate, and \( t \) is the time in years.

21. Suppose that an investment is governed by an accumulation function
    \[ A(t) = \frac{A_0}{1 - r} \]
    Where \( A_0 \) is the initial amount, \( r \) is the annual interest rate, and \( t \) is the time in years.

22. Suppose that an investment is governed by an accumulation function
    \[ A(t) = A_0 e^{rt} \]
    Where \( A_0 \) is the initial amount, \( r \) is the annual interest rate, and \( t \) is the time in years.

23. Suppose that an investment is governed by an accumulation function
    \[ A(t) = \frac{A_0}{1 - r} \]
    Where \( A_0 \) is the initial amount, \( r \) is the annual interest rate, and \( t \) is the time in years.

24. Suppose that an investment is governed by an accumulation function
    \[ A(t) = A_0 e^{rt} \]
    Where \( A_0 \) is the initial amount, \( r \) is the annual interest rate, and \( t \) is the time in years.

25. Suppose that an investment is governed by an accumulation function
    \[ A(t) = \frac{A_0}{1 - r} \]
    Where \( A_0 \) is the initial amount, \( r \) is the annual interest rate, and \( t \) is the time in years.