1. The stock of Jessica’s Diamonds Incorporated does not currently pay a dividend. However, it is anticipated that the stock will start to pay quarterly dividend at the end of five years. The first dividend is expected to be 3.15 and will be paid at the end of five years. Each subsequent dividend is expected to be 101% of the prior dividend. In other words, the dividend paid in 5 years and 3 months will be \((3.15)(1.01)\), the dividend paid in 5 years and 6 months will be \((3.15)(1.01)^2\), etc.

Using the dividend discount method, calculate the theoretical price of this stock in order to produce an annual effective yield of 12%.

Solution:

The first dividend will be 3.15 at time 5.
The second dividend will be 3.15(1.01) at time 5.25.
The third dividend will be 3.15(1.01)^2 at time 5.5.

\[ PV = 3.15v^5 + 3.15(1.01)v^{5.25} + 3.15(1.01)^2v^{5.5} + \ldots \]

\[ = \frac{3.15v^5 - 0}{1 - 1.01v^{0.25}} = \frac{3.15(1.12)^{-5}}{1 - \frac{1.01}{(1.12)^{0.25}}} = 98.13 \]
2. You are given the following spot interest rate curve:

<table>
<thead>
<tr>
<th>Time t</th>
<th>Spot Rate r_t</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.030</td>
</tr>
<tr>
<td>1.0</td>
<td>0.034</td>
</tr>
<tr>
<td>1.5</td>
<td>0.039</td>
</tr>
<tr>
<td>2.0</td>
<td>0.045</td>
</tr>
<tr>
<td>2.5</td>
<td>0.050</td>
</tr>
<tr>
<td>3.0</td>
<td>0.053</td>
</tr>
</tbody>
</table>

A two year bond matures for 10,000 and has a semi-annual coupons of 400.

Calculate the price of this bond.

**Solution:**

\[
P = \frac{400}{(1.030)^{0.5}} + \frac{400}{(1.034)^{1.0}} + \frac{400}{(1.039)^{1.5}} + \frac{20,400}{(1.045)^{2.0}} = 10,682.26
\]
3. You can purchase the following two bonds:

   a. A one year zero coupon bond which matures for 10,000 and has a price of 9500.
   
   b. A two year bond which matures for 20,000 and pays annual coupons of 2000. The price of the bond is 21,274.44.

Calculate the 2 year spot interest rate.

Solution:

\[ 9500 = (10,000)(1 + r_1)^{-1} \implies 1 + r_1 = \frac{10,000}{9500} = 1.052631579 \]

\[ 21,274.44 = 2000(1 + r_1)^{-1} + (2000 + 20,000)(1 + r_2)^{-2} \]

\[ 21,274.44 - 2000(0.95) = (22,000)(1 + r_2)^{-2} \]

\[ 19,374.44 = 22,000(1 + r_2)^{-2} \implies (1 + r_2)^2 = \frac{22,000}{19,374.44} \implies r_2 = \sqrt{\frac{22,000}{19,374.44}} - 1 = 0.0656063 \]
1. You are given the following spot interest rate curve:

<table>
<thead>
<tr>
<th>Time t</th>
<th>Spot Rate r_t</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.034</td>
</tr>
<tr>
<td>1.0</td>
<td>0.039</td>
</tr>
<tr>
<td>1.5</td>
<td>0.045</td>
</tr>
<tr>
<td>2.0</td>
<td>0.050</td>
</tr>
<tr>
<td>2.5</td>
<td>0.053</td>
</tr>
<tr>
<td>3.0</td>
<td>0.057</td>
</tr>
</tbody>
</table>

A two year bond matures for 20,000 and has a semi-annual coupons of 600.

Calculate the price of this bond.

Solution:

\[
P = \frac{600}{(1.034)^{0.5}} + \frac{600}{(1.039)^{1.0}} + \frac{600}{(1.045)^{1.5}} + \frac{20,600}{(1.05)^{2.0}} = 20,414.00
\]
2. You can purchase the following two bonds:
   
   a. A one year zero coupon bond which matures for 10,000 and has a price of 9600.
   
   b. A two year bond which matures for 20,000 and pays annual coupons of 2500. The price of the bond is 22,106.05.

   Calculate the 2 year spot interest rate.

Solution:

\[ 9600 = (10,000)(1 + r_1)^{-1} \Rightarrow 1 + r_1 = \frac{10,000}{9600} = 1.04166666 \]

\[ 22,106.05 = 2500(1 + r_1)^{-1} + (2500 + 20,000)(1 + r_2)^{-2} \]

\[ 22,106.05 - 2500(0.96) = (2500 + 20,000)(1 + r_2)^{-2} \]

\[ 19,706.05 = 22,500(1 + r_2)^{-2} \Rightarrow (1 + r_2)^2 = \frac{22,500}{19,706.05} \Rightarrow r_2 = \sqrt{\frac{22,500}{19,706.05}} - 1 = 0.068541684 \]
3. The stock of Jessica’s Diamonds Incorporated does not currently pay a dividend. However, it is anticipated that the stock will start to pay quarterly dividend at the end of five years. The first dividend is expected to be 5.15 and will be paid at the end of five years. Each subsequent dividend is expected to be 101% of the prior dividend. In other words, the dividend paid in 5 years and 3 months will be (5.15)(1.01), the dividend paid in 5 years and 6 months will be (5.15)(1.01)^2, etc.

Using the dividend discount method, calculate the theoretical price of this stock in order to produce an annual effective yield of 14%.

**Solution:**

The first dividend will be 5.15 at time 5.
The second dividend will be 5.15(1.01) at time 5.25.
The third dividend will be 5.15(1.01)^2 at time 5.5.
etc.

\[ PV = 5.15v^5 + 5.15(1.01)v^{5.25} + 5.15(1.01)^2v^{5.50} + \ldots \]

\[ = \frac{5.15v^5 - 0}{1 - 1.01v^{0.25}} = \frac{5.15(1.14)^5}{1 - \frac{1.01}{(1.14)^{0.25}}} = 118.62 \]
1. You can purchase the following two bonds:
   
   a. A one year zero coupon bond which matures for 10,000 and has a price of 9650.
   
   b. A two year bond which matures for 20,000 and pays annual coupons of 2200. The price of the bond is 21,599.92.

   Calculate the 2 year spot interest rate.

   **Solution:**

   \[
   9650 = (10,000)(1 + r_1)^{-1} \implies 1 + r_1 = \frac{10,000}{9650} = 1.03626943
   \]

   \[
   21,559.92 = 2200(1 + r_1)^{-1} + (2200 + 20,000)(1 + r_2)^{-2}
   \]

   \[
   21,559.92 - 2200(0.965) = 22,200(1 + r_2)^{-2}
   \]

   \[
   19,476.92 = 22,200(1 + r_2)^{-2} \implies (1 + r_2)^2 = \frac{22,200}{19,476.92} \implies r_2 = \sqrt{\frac{22,200}{19,476.92}} - 1 = 0.06761913
   \]
2. The stock of Jessica’s Diamonds Incorporated does not currently pay a dividend. However, it is anticipated that the stock will start to pay quarterly dividend at the end of five years. The first dividend is expected to be 4.15 and will be paid at the end of five years. Each subsequent dividend is expected to be 101% of the prior dividend. In other words, the dividend paid in 5 years and 3 months will be \((4.15)(1.01)\), the dividend paid in 5 years and 6 months will be \((4.15)(1.01)^2\), etc.

Using the dividend discount method, calculate the theoretical price of this stock in order to produce an annual effective yield of 10%.

**Solution:**

The first dividend will be 4.15 at time 5.
The second dividend will be 4.15(1.01) at time 5.25.
The third dividend will be 4.15(1.01)^2 at time 5.5.

\[
PV = 4.15v^5 + 4.15(1.01)v^{5.25} + 4.15(1.01)^2v^{5.50} + ... 
\]

\[
= \frac{4.15v^5 - 0}{1 - 1.01^{0.25}} = \frac{4.15(1.10)^{-5}}{1 - \frac{1.01}{(1.10)^{0.25}}} = 186.98
\]
3. You are given the following spot interest rate curve:

<table>
<thead>
<tr>
<th>Time t</th>
<th>Spot Rate r_t</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.036</td>
</tr>
<tr>
<td>1.0</td>
<td>0.039</td>
</tr>
<tr>
<td>1.5</td>
<td>0.044</td>
</tr>
<tr>
<td>2.0</td>
<td>0.048</td>
</tr>
<tr>
<td>2.5</td>
<td>0.051</td>
</tr>
<tr>
<td>3.0</td>
<td>0.057</td>
</tr>
</tbody>
</table>

A two year bond matures for 20,000 and has a semi-annual coupons of 700.

Calculate the price of this bond.

**Solution:**

\[
P = \frac{700}{(1.036)^{0.5}} + \frac{700}{(1.039)^{1.0}} + \frac{700}{(1.044)^{1.5}} + \frac{20,700}{(1.048)^{2.0}} = 20,864.91
\]
1. You are given the following spot interest rate curve:

<table>
<thead>
<tr>
<th>Time t</th>
<th>Spot Rate r_t</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>0.040</td>
</tr>
<tr>
<td>1.0</td>
<td>0.044</td>
</tr>
<tr>
<td>1.5</td>
<td>0.049</td>
</tr>
<tr>
<td>2.0</td>
<td>0.054</td>
</tr>
<tr>
<td>2.5</td>
<td>0.057</td>
</tr>
<tr>
<td>3.0</td>
<td>0.059</td>
</tr>
</tbody>
</table>

A two year bond matures for 8,000 and has a semi-annual coupons of 350.

Calculate the price of this bond.

**Solution:**

\[
P = \frac{350}{(1.040)^{0.5}} + \frac{350}{(1.044)^{1.0}} + \frac{350}{(1.049)^{1.5}} + \frac{8,350}{(1.054)^{2.0}} = 8,520.54
\]
2. The stock of Jessica's Diamonds Incorporated does not currently pay a dividend. However, it is anticipated that the stock will start to pay quarterly dividend at the end of five years. The first dividend is expected to be 3.50 and will be paid at the end of five years. Each subsequent dividend is expected to be 101\% of the prior dividend. In other words, the dividend paid in 5 years and 3 months will be (3.50)(1.01), the dividend paid in 5 years and 6 months will be \((3.50)(1.01)^2\), etc.

Using the dividend discount method, calculate the theoretical price of this stock in order to produce an annual effective yield of 16%.

**Solution:**

The first dividend will be 3.50 at time 5.
The second dividend will be 3.50(1.01) at time 5.25.
The third dividend will be 3.50(1.01)^2 at time 5.5.

\[ PV = 3.50v^5 + 3.50(1.01)v^{5.25} + 3.50(1.01)^2v^{5.50} + ... \]

\[ = \frac{3.50v^5 - 0}{1 - 1.01^{0.25}} = \frac{3.50(1.16)^{-5}}{1 - \frac{1.01}{(1.16)^{0.25}}} = 62.20 \]
3. You can purchase the following two bonds:
   
a. A one year zero coupon bond which matures for 10,000 and has a price of 9750.

b. A two year bond which matures for 20,000 and pays annual coupons of 2000. The price of the bond is 21,312.68.

Calculate the 2 year spot interest rate.

Solution:

\[
9750 = (10,000)(1 + r_1)^{-1} \implies 1 + r_1 = \frac{10,000}{9750} = 1.0256410
\]

\[
21,312.68 = 2000(1 + r_1)^{-1} + (2000 + 20,000)(1 + r_2)^{-2}
\]

\[
21,312.68 - 2000(0.975) = (22,000)(1 + r_2)^{-2}
\]

\[
19,362.68 = 22,000(1 + r_2)^{-2} \implies (1 + r_2)^2 = \frac{22,000}{19,362.68} \implies r_2 = \sqrt{\frac{22,000}{19,362.68}} - 1 = 0.06593
\]