There are 3 versions. Find your version.
1. Yujie has the option of purchasing the following two bonds.
   
   i. Bond A is a zero coupon bond with a maturity value of 100,000 at the end of 30 years.
   
   ii. Bond B is a 10 year bond with semi-annual coupons and a maturity value of 25,000.

Both bonds have a price of \( P \) if they are purchased to yield a rate of 5% convertible semi-annually.

Calculate the amount of the semi-annual coupon for Bond B.

**Solution:**

Bond A:

\[(1.025)^2 = (1+i) \rightarrow i = 0.050625 \Rightarrow P = 100,000(1.050625)^{-30} = 22,728.359\]

Bond B:

\[22,728.359 = (coupon) \cdot a_{20|0.025} + 25,000(1.025)^{-20}\]

\[coupon = \frac{22,728.359 - 25,000(1.025)^{-20}}{1 - (1.025)^{-20}} = 479.28\]
2. KC is the beneficiary of a trust that is paying her a continuous annuity over the next 25 years. The annuity pays at a rate of $500t$ at time $t$.

Using an annual effective interest rate of 7%, calculate the present value of this annuity.

**Solution:**

\[
\delta = \ln(1 + i) \implies \delta = \ln(1.07) = 0.067659
\]

\[
PV = 500 \left( \frac{1 - e^{-25\delta}}{\delta} - 25e^{-25\delta} \right) = 55,060.44
\]
3. Madison is the winner of the Lottery. She can take her winnings by choosing one of the following options:

i. An annuity due with monthly payments for 20 years. Each monthly payment in the first year is 10,000. Each monthly payment in the second year is 20,000. Each monthly payment in the third year is 30,000. This pattern continues until each monthly payment in the 20th year is 200,000.

ii. A lump sum payment of $X$.

At an annual effective interest rate of 10%, these two options have the same present value.

Calculate $X$.

Solution:

\[
1.10 = \left(1 + \frac{i^{(12)}}{12}\right)^{12} \rightarrow \frac{i^{(12)}}{12} = 0.00797941
\]

\[
X = 1.00797941 \left[ \frac{10,000 \left[ \frac{1 - (1.10)^{-20}}{0.10} \right]}{0.00797941} - 20(1.10)^{-20} \right] = 8,079,891.09
\]
4. A bond has a book value right after the 13th coupon of 10,000. The bond pays semi-annual coupons of 280. The bond was purchased to yield 5% convertible semi-annually.

Calculate the principal in the 16th coupon.

**Solution:**

\[ B_{13} = 10,000 \]
\[ I_{14} = 0.025B_{13} = 250 \]
\[ P_{14} = 280 - I_{14} = 30 \]
\[ B_{14} = B_{13} - P_{14} = 9,970 \]
\[ I_{15} = 0.025B_{14} = 249.25 \]
\[ P_{15} = 280 - I_{15} = 30.75 \]
\[ B_{15} = B_{14} - P_{15} = 9,939.25 \]
\[ I_{16} = 0.025B_{15} = 248.481 \]
\[ P_{16} = 280 - I_{16} = 31.519 \]

*Or*

\[ P_{16} = P_{14}(1+i)^2 = (30)(1.025)^2 = 31.51875 \]
5. Ayanna purchases a special 30 year bond which matures for 200,000. The bond has annual coupons that increase. The first coupon is for 1000. The second coupon is for 2000. The coupons continue to increase until a coupon of 30,000 is paid in the 30th year.

Ayanna purchases the bond to yield an annual effective yield of 8%.

Calculate the price that Ayanna paid for this bond.

**Solution:**

\[
P = 1000a_{90} + \frac{1000}{0.08}[a_{90} - 30(1.08)^{-30}] + 200,000(1.08)^{-30}
\]

\[
P = 134,589.04
\]
6. A loan of \( L \) is being repaid with level annual payments of 1000 for \( n \) years. The interest in the 10\textsuperscript{th} payment is 678.11. The interest in the 18\textsuperscript{th} payment is 467.27.

Determine \( L \), the amount of the loan.

\textbf{Solution:}

\[ P_{10} = PMT - I_{10} = 1,000 - 678.11 = 321.89 \]
\[ P_{18} = PMT - I_{18} = 1,000 - 467.27 = 532.73 \]

\[ 321.89(1+i)^8 = 532.73 \]

\[ \rightarrow i = 0.065 \]

\[ P_1(1.065)^9 = 321.89 \]

\[ \therefore P_1 = 182.625 \]

\[ I_1 = 1,000 - 182.625 = 817.375 \]

\[ I_1 = i \cdot L = (0.065)L \]

\[ \therefore L = \frac{817.375}{0.065} = 12,575 \]
7. Ethan deposits 5,600 at the end of each year for 15 years into the Farlow Fund. The Farlow Fund pays an annual effective interest rate of 6.25%.

At the end of each year, Ethan removes any interest earned from the Farlow Fund and deposits it into Ballard Bank. Ballard Bank pays interest at an annual effective rate of 8%.

At the end of 15 years, Ethan withdraws all his money from both the Farlow Fund and Ballard Bank. Determine the total amount that Ethan withdraws.

**Solution:**

\[ I = (0.0625 \times 5600) = 350 \]

Amt in Farlow Fund = (15 \times 5600) = 84,000

Amt in Ballard Bank:

\[ P = 350, \quad Q = 350, \quad i = 0.08, \quad N = 14 \]

\[ PV = 350 \times a_{[i]} + \frac{350}{0.08} \left[ a_{[i]} - 14(1.08)^{-14} \right] = 18,100.78 \]

\[ AV = PV \times (1.08)^{14} = 53,165.50 \]

Total = 84,000 + 53,165.50 = 137,165.50
8. A loan is being repaid with monthly payments for 10 years. The payments are increasing payments with the first payment of 250. The second payment is 250(1.005). The third payment is 250(1.005)^2. The payment continues to increase in the same pattern until the last payment is 250(1.005)^119.

The interest rate on the loan is 12% compounded monthly.

Find the principal in the payment made at the end of the 118th month.

Solution:

\[
\frac{i(12)}{12} = \frac{0.12}{12} = 0.01 \quad N = 10 \cdot 12 = 120
\]

\[OLB_{117} = \text{Present Value of Future Cash Flows} = 250(1.005)^{117}(1.01)^{-1} + 250(1.005)^{118}(1.01)^{-2} + 250(1.005)^{119}(1.01)^{-3} = 1,324.395\]

\[\text{Principle}_{18} = \text{Payment}_{18} - I_{18} = 250(1.005)^{117} - (1,324.395)(0.01) = 434.85\]
9. Emily needs to borrow 500,000 to build a factory for Whisler's Outstanding Whistles (WOW!). She has the choice of the following two loans:

   a. An 10 year amortization loan with annual payments. The interest rate on the loan is an annual effective rate of 7.10%.

   b. A 10 year sinking fund loan with annual payments. At the end of each year, Emily will pay interest at a rate of 6%. Additionally, at the end of each year, Emily will make a deposit into a sinking fund earning 4% interest. The deposit is determined such that at the end of 10 years, the amount in the sinking fund will exactly repay the amount of the loan.

Which loan should Emily select? Explain why!

Solution:

(a) \[ PV = 500,000 \quad N=10 \quad I/Y=7.1\% \]
\[ \therefore PMT=71,518.177 \]

(b) \[ I = iL = (0.06)500,000 = 30,000 \]
\[ j = 0.04 \]

\[ D = \frac{L}{S_{10,0.04}} = 41,645.472 \]

\[ P=I+D=71,645.472 \]

Emily should choose loan (a) because it has lower payments.
10. Courtney has $X$ to invest. She can invest in the following two options:

a. A 20 year bond that has a par value of 100,000 with a coupon rate of 8.2% convertible semi-annually. The bond matures for 115,000. The price of the bond is $X$ when purchased to yield 6.4% convertible semi-annually.

b. An annuity due with annual payments for 10 years. The first payment is $P$, the second payment is $P + 2000$, and the third payment is $P + 4000$. The payments continue to increases in the same pattern until the last payment of $P + 18,000$ is paid. Using an annual effective interest rate of 7%, the present value of this annuity is $X$.

Calculate $P$.

**Solution:**

\[
\frac{r}{2} = \frac{0.082}{2} = 0.041 \quad i = \frac{0.064}{2} = 0.032 \quad N = 20 \times 2 = 40
\]

\[
Fr = 100,000(0.041) = 4100
\]

\[
X = 4100a_{40|0.032} + 115,000(1.032)^{-40} = 124,401.84
\]

\[
X = (1.07) \left[ Pa_{10|0.07} + \frac{2000}{0.07} \left( a_{10|0.07} - 10(1.07)^{-10} \right) \right] = 124,401.84
\]

\[
\therefore P = 8,661.15
\]
11. A 20 year bond with a coupon rate of 8% convertible semi-annually has a maturity value of $C$. The par value of the bond is 90% of $C$. The bond is purchased for 11,630.08 to yield 7.5% convertible semi-annually.

Determine the amount of premium or discount. Be sure to state whether it is a premium or a discount.

**Solution:**

\[ r = \frac{0.08}{2} = 0.04 \quad i = \frac{0.075}{2} = 0.0375 \quad N = 20 \cdot 2 = 40 \]

\[ Fr = 0.9C(0.04) = 0.036C \]

\[ 11,630.08 = 0.036C a_{40.0375} + C(1.0375)^{-40} \]

\[ \therefore C = 11,999.998 \approx 12,000 \]

\[ C > P \rightarrow discount \]

\[ discount = C - P = 12,000 - 11,630.08 = 369.92 \]
12. A 20 year callable bond has a par value of 25,000. The bond matures for par and has a coupon rate of 6% convertible semi-annually.

Additionally, the bond can be called at the times and amounts shown in the table:

<table>
<thead>
<tr>
<th>Time</th>
<th>Call Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 Years</td>
<td>27,200</td>
</tr>
<tr>
<td>14 Years</td>
<td>26,500</td>
</tr>
<tr>
<td>16 Years</td>
<td>25,900</td>
</tr>
<tr>
<td>18 Years</td>
<td>25,400</td>
</tr>
</tbody>
</table>

Calculate the price of this bond to assure a yield of 5% convertible semi-annually.

Solution:

<table>
<thead>
<tr>
<th>N</th>
<th>I/Y</th>
<th>Coupon</th>
<th>FV</th>
<th>PV</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>2.5</td>
<td>15,000(0.03)=750</td>
<td>27,200</td>
<td>28,451.95</td>
</tr>
<tr>
<td>28</td>
<td>2.5</td>
<td>750</td>
<td>26,500</td>
<td>28,246.93</td>
</tr>
<tr>
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<td>2.5</td>
<td>750</td>
<td>25,900</td>
<td>28,139.54</td>
</tr>
<tr>
<td>36</td>
<td>2.5</td>
<td>750</td>
<td>25,400</td>
<td><strong>28,108.97</strong></td>
</tr>
<tr>
<td>40</td>
<td>2.5</td>
<td>750</td>
<td>25,000</td>
<td>28,137.85</td>
</tr>
</tbody>
</table>

28,108.97 is the lowest, so that will be the price.
1. A 20 year callable bond has a par value of 25,000. The bond matures for par and has a coupon rate of 6% convertible semi-annually.

Additionally, the bond can be called at the times and amounts shown in the table:

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<td>25,800</td>
</tr>
<tr>
<td>18 Years</td>
<td>25,400</td>
</tr>
</tbody>
</table>

Calculate the price of this bond to assure a yield of 5% convertible semi-annually.

Solution:

<table>
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<td>28,246.93</td>
</tr>
<tr>
<td>32</td>
<td>2.5</td>
<td>750</td>
<td>25,800</td>
<td>28,094.16</td>
</tr>
<tr>
<td>36</td>
<td>2.5</td>
<td>750</td>
<td>25,400</td>
<td>28,108.97</td>
</tr>
<tr>
<td>40</td>
<td>2.5</td>
<td>750</td>
<td>25,000</td>
<td>28,137.85</td>
</tr>
</tbody>
</table>

28,094.16 is the lowest, so that will be the price.
2. A 20 year bond with a coupon rate of 8% convertible semi-annually has a maturity value of $C$. The par value of the bond is 90% of $C$. The bond is purchased for $11,630.08 to yield 7.5% convertible semi-annually.

Determine the amount of premium or discount. Be sure to state whether it is a premium or a discount.

Solution:

\[
r = \frac{0.08}{2} = 0.04 \quad i = \frac{0.075}{2} = 0.0375
\]

\[N = 20 \cdot 2 = 40\]

\[Fr = 0.9C(0.04) = 0.036C\]

\[11630.08 = 0.036C \cdot a_{40} + C(1.0375)^{40}\]

\[\therefore C = 11,999.998 \approx 12,000\]

\[C > P \rightarrow discount\]

\[discount = C - P = 12,000 - 11,630.08 = 369.92\]
3. Courtney has $X$ to invest. She can invest in the following two options:

   a. A 20 year bond that has a par value of 100,000 with a coupon rate of 8.4% convertible semi-annually. The bond matures for 115,000. The price of the bond is $X$ when purchased to yield 6.2% convertible semi-annually.

   b. An annuity due with annual payments for 10 years. The first payment is $P$, the second payment is $P + 2000$, and the third payment is $P + 4000$. The payments continue to increases in the same pattern until the last payment of $P + 18,000$ is paid. Using an annual effective interest rate of 8%, the present value of this annuity is $X$.

Calculate $P$.

Solution:

\[
r = \frac{0.084}{2} = 0.042 \quad i = \frac{0.062}{2} = 0.031 \quad N = 20 \times 2 = 40
\]

\[Fr = 100,000(0.042) = 4,200\]

\[X = 4,200a_{40} + 115,000(1.031)^{-40} = 129,443.475\]

\[X = (1.08)[Pa_{10} + \frac{2000}{0.08}(a_{10} - 10(1.08)^{-10})] = 129,433.475\]

\[\therefore P = 10,119.31\]
4. Emily needs to borrow $500,000 to build a factory for Whisler’s Outstanding Whistles (WOW!). She has the choice of the following two loans:

   a. An 10 year amortization loan with annual payments. The interest rate on the loan is an annual effective rate of 7.10%.

   b. A 10 year sinking fund loan with annual payments. At the end of each year, Emily will pay interest at a rate of 6%. Additionally, at the end of each year, Emily will make a deposit into a sinking fund earning 4% interest. The deposit is determined such that at the end of 10 years, the amount in the sinking fund will exactly repay the amount of the loan.

Which loan should Emily select? Explain why!

Solution:

(a) \( PV = 500,000 \quad N=10 \quad I/Y=7.1\% \)

\[ \therefore PMT=71,518.177 \]

(b) \( I = iL = (0.06)500,000 = 30,000 \)

\( j=0.04 \)

\[ D=\frac{L}{S_{10|0.04}}=41,645.472 \]

\[ P = I + D = 71,645.472 \]

Emily should choose loan (a) because it has lower payments.
5. A loan is being repaid with monthly payments for 10 years. The payments are increasing payments with the first payment of 250. The second payment is 250(1.007). The third payment is 250(1.007)^2. The payment continue to increase in the same pattern until the last payment is 250(1.007)^{119}.

The interest rate on the loan is 15% compounded monthly.

Find the principal in the payment made at the end of the 118th month.

Solution:

\[ \frac{r}{12} = \frac{0.15}{12} = 0.0125 \quad N = 10 \times 12 = 120 \]

\( OLB_{117} = \text{Present Value of Future Cash Flows} = \)

\( 250(1.007)^{117}(1.0125)^{-1} + 250(1.007)^{118}(1.0125)^{-2} + 250(1.007)^{119}(1.0125)^{-3} = 1,666.30 \)

\( \text{Principle}_{18} = \text{Payment}_{18} - I_{18} = 250(1.007)^{117} - (1666.30)(0.0125) = 544.613 \)
6. Ethan deposits 6,500 at the end of each year for 15 years into the Farlow Fund. The Farlow Fund pays an annual effective interest rate of 6.75%.

At the end of each year, Ethan removes any interest earned from the Farlow Fund and deposits it into Ballard Bank. Ballard Bank pays interest at an annual effective rate of 8%.

At the end of 15 years, Ethan withdraws all his money from both the Farlow Fund and Ballard Bank. Determine the total amount that Ethan withdraws.

Solution:

\[ I = (0.0675)(6500) = 438.75 \]

Amt in Farlow Fund = (15)6500 = 97,500

Amt in Ballard Bank:
P = 438.75  Q = 438.75  i = 0.08  N = 14

\[ PV = 438.75a_{14|} + \frac{438.75}{0.08} [a_{14|} - 14(1.08)^{-14}] = 22,690.622 \]

\[ AV = PV(1.08)^{14} = 66,646.75 \]

Total = 97,500 + 66,646.75 = 164,146.75
7. A loan of $L$ is being repaid with level annual payments of 1000 for $n$ years. The interest in the 10th payment is 678.11. The interest in the 18th payment is 467.27.

Determine $L$, the amount of the loan.

Solution:

\[
P_{10} = PMT - I_{10} = 1,000 - 678.11 = 321.89
\]
\[
P_{18} = PMT - I_{18} = 1,000 - 467.27 = 532.73
\]
\[
321.89(1 + i)^8 = 532.73
\]
\[
\rightarrow i = 0.065
\]

\[
P_1(1.065)^9 = 321.89
\]
\[
\therefore P_1 = 182.625
\]

\[
I_1 = 1,000 - 182.625 = 817.375
\]
\[
I_1 = i \cdot L = (0.065)L
\]
\[
\therefore L = \frac{817.375}{0.065} = 12,575
\]
8. Ayanna purchases a special 30 year bond which matures for 200,000. The bond has annual coupons that increase. The first coupon is for 1000. The second coupon is for 2000. The coupons continue to increase until a coupon of 30,000 is paid in the 30th year.

Ayanna purchases the bond to yield an annual effective yield of 9%.

Calculate the price that Ayanna paid for this bond.

**Solution:**

\[
P = 1000a_{30} + \frac{1000}{0.09} [a_{30} - 30(1.09)^{-30}] + 200,000(1.09)^{-30}
\]

\[
P = 114,375.88
\]
9. A bond has a book value right after the 13th coupon of 10,000. The bond pays semi-annual coupons of 280. The bond was purchased to yield 5% convertible semi-annually.

Calculate the principal in the 16th coupon.

**Solution:**

\[ B_{13} = 10,000 \]
\[ I_{14} = 0.025 \times B_{13} = 250 \]
\[ P_{14} = 280 - I_{14} = 30 \]
\[ B_{14} = B_{13} - P_{14} = 9,970 \]
\[ I_{15} = 0.025 \times B_{14} = 249.25 \]
\[ P_{15} = 280 - I_{15} = 30.75 \]
\[ B_{15} = B_{14} - P_{15} = 9,939.25 \]
\[ I_{16} = 0.025 \times B_{15} = 248.481 \]
\[ P_{16} = 280 - I_{16} = 31.519 \]

Or

\[ P_{16} = P_{14}(1+i)^2 = (30)(1.025)^2 = 31.51875 \]
10. Madison is the winner of the Lottery. She can take her winnings by choosing one of the following options:

i. An annuity due with monthly payments for 20 years. Each monthly payment in the first year is 10,000. Each monthly payment in the second year is 20,000. Each monthly payment in the third year is 30,000. This pattern continues until each monthly payment in the 20th year is 200,000.

ii. A lump sum payment of $X$.

At an annual effective interest rate of 9%, these two options have the same present value.

Calculate $X$.

**Solution:**

\[
1.09 = \left(1 + \frac{i^{(12)}}{12}\right)^{12} \Rightarrow \frac{i^{(12)}}{12} = 0.0072073
\]

\[
X = 1.0072073 \times \frac{10,000\left(\bar{a}_{20|0.09} - 20(1.09)^{-20}\right)}{0.0072073} =
\]

\[
1.0072073 \times \frac{10,000\left(1-(1.09)^{-20}\right)0.09}{0.0072073(1.09) - 20(1.09)^{-20}} = 8,917,999.781
\]
11. KC is the beneficiary of a trust that is paying her a continuous annuity over the next 25 years. The annuity pays at a rate of $600t$ at time $t$.

Using an annual effective interest rate of 6%, calculate the present value of this annuity.

**Solution:**

\[
\delta = \ln(1 + i) \implies \delta = \ln(1.06) = 0.0582689
\]

\[
Pv = 600 \left( \frac{1 - e^{-25\delta}}{\delta} - 25e^{-25\delta} \right) = 75,561.74
\]
12. Yujie has the option of purchasing the following two bonds.

i. Bond A is a zero coupon bond with a maturity value of 100,000 at the end of 30 years.

ii. Bond B is a 10 year bond with semi-annual coupons and a maturity value of 25,000.

Both bonds have a price of $P$ if they are purchased to yield a rate of 4.8% convertible semi-annually.

Calculate the amount of the semi-annual coupon for Bond B.

**Solution:**

Bond A:

$$(1.04)^2 = (1 + i) \rightarrow i = 0.048576 \Rightarrow P = 100,000(1.048576)^{-30} = 24,099.199$$

Bond B:

$$24,099.199 = \text{(coupon)}a_{20} + 25,000(1.04)^{-20}$$

$$\text{coupon} = \frac{24,099.199 - 25,000(1.04)^{-20}}{1 - (1.04)^{-20}} = \frac{542.761}{0.024}$$
Math 373
Test 2
Spring 2016
March 8, 2016

1. A 20 year callable bond has a par value of 25,000. The bond matures for par and has a coupon rate of 6% convertible semi-annually.

Additionally, the bond can be called at the times and amounts shown in the table:

<table>
<thead>
<tr>
<th>Time</th>
<th>Call Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>12 Years</td>
<td>26,950</td>
</tr>
<tr>
<td>14 Years</td>
<td>26,250</td>
</tr>
<tr>
<td>16 Years</td>
<td>25,900</td>
</tr>
<tr>
<td>18 Years</td>
<td>25,500</td>
</tr>
</tbody>
</table>

Calculate the price of this bond to assure a yield of 5% convertible semi-annually.

Solution:

<table>
<thead>
<tr>
<th>N</th>
<th>I/Y</th>
<th>Coupon</th>
<th>FV</th>
<th>PV</th>
</tr>
</thead>
<tbody>
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<td>24</td>
<td>2.5</td>
<td>15,000(0.03)=750</td>
<td>26,950</td>
<td>28,313.73</td>
</tr>
<tr>
<td>28</td>
<td>2.5</td>
<td>750</td>
<td>26,250</td>
<td><strong>28,121.71</strong></td>
</tr>
<tr>
<td>32</td>
<td>2.5</td>
<td>750</td>
<td>25,900</td>
<td>28,139.54</td>
</tr>
<tr>
<td>36</td>
<td>2.5</td>
<td>750</td>
<td>25,500</td>
<td>28,150.08</td>
</tr>
<tr>
<td>40</td>
<td>2.5</td>
<td>750</td>
<td>25,000</td>
<td>28,137.85</td>
</tr>
</tbody>
</table>

28,121.71 is the lowest, so that will be the price.
2. KC is the beneficiary of a trust that is paying her a continuous annuity over the next 25 years. The annuity pays at a rate of $800e^{\delta}$ at time $t$.

Using an annual effective interest rate of 5%, calculate the present value of this annuity.

**Solution:**

\[
\delta = \ln(1+i) \implies \delta = \ln(1.05) = 0.0487901
\]

\[
PV = 800 \left[ \frac{1-e^{-25\delta}}{\delta} - \frac{25e^{-25\delta}}{\delta} \right] = 115,775.12
\]
3. Ayanna purchases a special 30 year bond which matures for 200,000. The bond has annual coupons that increase. The first coupon is for 1000. The second coupon is for 2000. The coupons continue to increase until a coupon of 30,000 is paid in the 30th year.

Ayanna purchases the bond to yield an annual effective yield of 7%.

Calculate the price that Ayanna paid for this bond.

Solution:

\[ P = 1000a_{30} + \frac{1000}{0.07} [a_{30} - 30(1.07)^{-30}] + 200,000(1.07)^{-30} = 159,654.29 \]
4. A loan of $L$ is being repaid with level annual payments of 1000 for $n$ years. The interest in the 10th payment is 678.11. The interest in the 18th payment is 467.27.

Determine $L$, the amount of the loan.

**Solution:**

\[ P_{10} = PMT - I_{10} = 1,000 - 678.11 = 321.89 \]
\[ P_{18} = PMT - I_{18} = 1,000 - 467.27 = 532.73 \]
\[ 321.89(1 + i)^8 = 532.73 \]
\[ \rightarrow i = 0.065 \]

\[ P_1(1.065)^9 = 321.89 \]
\[ \therefore P_1 = 182.625 \]

\[ I_1 = 1,000 - 182.625 = 817.375 \]
\[ I_1 = i \cdot L = (0.065)L \]
\[ \therefore L = \frac{817.375}{0.065} = 12,575 \]
5. A 20 year bond with a coupon rate of 8% convertible semi-annually has a maturity value of \( C \). The par value of the bond is 90% of \( C \). The bond is purchased for 11,630.08 to yield 7.5% convertible semi-annually.

Determine the amount of premium or discount. Be sure to state whether it is a premium or a discount.

**Solution:**

\[
\begin{align*}
    r &= \frac{0.08}{2} = 0.04 \\
    i &= \frac{0.075}{2} = 0.0375 \\
    N &= 20 \cdot 2 = 40 \\
    Fr &= 0.9C(0.04) = 0.036 C \\
    11630.08 &= 0.036 C \frac{1}{a_{40}} + C (1.0375)^{-40} \\
    \therefore C &= 11,999.998 \approx 12,000 \\
    C > P \rightarrow discount \\
    discount &= C - P = 12,000 - 11,630.08 = 369.92
\end{align*}
\]
6. Courtney has $X$ to invest. She can invest in the following two options:

a. A 20 year bond that has a par value of 100,000 with a coupon rate of 8.6% convertible semi-annually. The bond matures for 115,000. The price of the bond is $X$ when purchased to yield 5.8% convertible semi-annually.

b. An annuity due with annual payments for 10 years. The first payment is $P$, the second payment is $P + 2000$, and the third payment is $P + 4000$. The payments continue to increases in the same pattern until the last payment of $P + 18,000$ is paid. Using an annual effective interest rate of 9%, the present value of this annuity is $X$.

Calculate $P$.

**Solution:**

$$r = \frac{0.086}{2} = 0.043 \quad i = \frac{0.058}{2} = 0.029 \quad N = 20 \cdot 2 = 40$$

$$Fr = 100,000(0.043) = 4,300$$

$$X = 4,300a_{40 \mid} + 115,000(1.029)^{-40} = 137,670.773$$

$$X = (1.09) \left[ Pa_{40 \mid} + \frac{2000}{0.09} (a_{40 \mid} - 10(1.09)^{-10}) \right] = 137,670.773$$

$$\therefore P = 12,085.08$$
7. Ethan deposits 3300 at the end of each year for 14 years into the Farlow Fund. The Farlow Fund pays an annual effective interest rate of 7.25%.

At the end of each year, Ethan removes any interest earned from the Farlow Fund and deposits it into Ballard Bank. Ballard Bank pays interest at an annual effective rate of 8%.

At the end of 14 years, Ethan withdraws all his money from both the Farlow Fund and Ballard Bank. Determine the total amount that Ethan withdraws.

**Solution:**

\[ I = (0.0725)3300 = 239.25 \]

Amt in Farlow Fund=\((14)3300=46,200\)

Amt in Ballard Bank:

\[ P=239.25 \quad Q=239.25 \quad i=0.08 \quad N=13 \]

\[ PV=239.25a_{13i} + \frac{239.25}{0.08} [a_{13i} - 13(1.08)^{-13}] = 10,400.743 \]

\[ AV = PV(1.08)^{13} = 30,548.996 \]

\[ Total = 46,200 + 30,548.996 = 76,749.00 \]
8. Madison is the winner of the Lottery. She can take her winnings by choosing one of the following options:

i. An annuity due with monthly payments for 20 years. Each monthly payment in the first year is 10,000. Each monthly payment in the second year is 20,000. Each monthly payment in the third year is 30,000. This pattern continues until each monthly payment in the 20th year is 200,000.

ii. A lump sum payment of $X$.

At an annual effective interest rate of 11%, these two options have the same present value.

Calculate $X$.

**Solution:**

\[1.11 = \left(1 + \frac{i^{(12)}}{12}\right)^{12} \rightarrow \frac{i^{(12)}}{12} = 0.0087346\]

\[X = 1.0087346 \left[ \frac{10,000(\bar{a}_{-20}) - 20(1.11)^{-20}}{0.0087346} \right] = 7,343,393.55\]
9. A bond has a book value right after the 13th coupon of 10,000. The bond pays semi-annual coupons of 280. The bond was purchased to yield 5% convertible semi-annually.

Calculate the principal in the 16th coupon.

Solution:

\[ B_{13} = 10,000 \]
\[ I_{14} = 0.025B_{13} = 250 \]
\[ P_{14} = 280 - I_{14} = 30 \]
\[ B_{14} = B_{13} - P_{14} = 9,970 \]
\[ I_{15} = 0.025B_{14} = 249.25 \]
\[ P_{15} = 280 - I_{15} = 30.75 \]
\[ B_{15} = B_{14} - P_{15} = 9,939.25 \]
\[ I_{16} = 0.025B_{15} = 248.481 \]
\[ P_{16} = 280 - I_{16} = 31.519 \]

Or

\[ P_{16} = P_{14}(1 + i)^2 = (30)(1.025)^2 = 31.51875 \]
10. A loan is being repaid with monthly payments for 10 years. The payments are increasing payments with the first payment of 250. The second payment is 250(1.004). The third payment is 250(1.004)^2. The payment continue to increase in the same pattern until the last payment is 250(1.004)^{119}.

The interest rate on the loan is 9% compounded monthly.

Find the principal in the payment made at the end of the 118th month.

**Solution:**

\[
\frac{i^{(12)}}{12} = \frac{0.09}{12} = 0.0075 \quad N = 10 \cdot 12 = 120
\]

\[
OLB_{117} = \text{Present Value of Future Cash Flows} = 250(1.004)^{117}(1.0075)^{-1} + 250(1.004)^{118}(1.0075)^{-2} + 250(1.004)^{119}(1.0075)^{-3} = 1,183.453
\]

\[
\text{Principle}_{18} = \text{Payment}_{18} - I_{18} = 250(1.004)^{117} - (1,324.395)(0.075) = 389.95
\]
11. Emily needs to borrow 500,000 to build a factory for Whisler’s Outstanding Whistles (WOW!). She has the choice of the following two loans:

a. An 10 year amortization loan with annual payments. The interest rate on the loan is an annual effective rate of 7.10%.

b. A 10 year sinking fund loan with annual payments. At the end of each year, Emily will pay interest at a rate of 6%. Additionally, at the end of each year, Emily will make a deposit into a sinking fund earning 4% interest. The deposit is determined such that at the end of 10 years, the amount in the sinking fund will exactly repay the amount of the loan.

Which loan should Emily select? Explain why!

**Solution:**

(a) \[ PV = 500,000 \quad N=10 \quad I/Y=7.1\% \]
\[ \therefore PMT=71,518.177 \]

(b) \[ I = (0.06)500,000 = 30,000 \]

\[ j = 0.04 \]

\[ D = \frac{L}{S_{10,0.04}} = 41,645.472 \]

\[ P=I+D=71,645.472 \]

Emily should choose loan (a) because it has lower payments.
12. Yujie has the option of purchasing the following two bonds.

   i. Bond A is a zero coupon bond with a maturity value of 100,000 at the end of 30 years.

   ii. Bond B is a 10 year bond with semi-annual coupons and a maturity value of 25,000.

Both bonds have a price of $P$ if they are purchased to yield a rate of 5.2% convertible semi-annually.

Calculate the amount of the semi-annual coupon for Bond B.

**Solution:**

**Bond A:**

$$(1.026)^2 = (1+i) \rightarrow i = 0.052676$$

$$P = 100,000(1.052676)^{-30} = 21,436.7209$$

**Bond B:**

$$21,436.7209 = (\text{coupon})a_{20|} + 25,000(1.026)^{-20}$$

$$\text{coupon} = \frac{21,436.7209 - 25,000(1.026)^{-20}}{1 - (1.026)^{-20}} = 419.26$$