Chapter 2, Section 2

1. JT invests $3000 in an account earning interest at an annual effective rate of 6%. How much will JT have at the end of three and one half years?

   \[\text{Amount} = 3000(1.06)^{3.5} = 3678.68\]

2. Elsa invests $3000 in an account earning a nominal interest rate of 6% compounded monthly. At the end of \(X\) months, Elsa has $3644.16. Determine \(X\).

   \[3000\left(1 + \frac{0.06}{12}\right)^x = 3644.16 \implies \left(1 + \frac{0.06}{12}\right)^x = 1.21472\]

   \[X \left[ \ln \left(1 + \frac{0.06}{12}\right) \right] = \ln(1.21472) \implies X = \frac{\ln(1.21472)}{\ln(1 + \frac{0.06}{12})} = 39 \text{ months}\]

3. Raza wants to have $5000 at the end of four years when he graduates in order help him move to the location of his new job. Raza invests \(P\) today into an account earning an annual effective interest rate equivalent to a nominal discount rate of 7% convertible quarterly.

   Determine the minimum \(P\) such that Raza will have $5000 in four years.

   \[P \left(1 - \frac{0.07}{4}\right)^{44} = 5000 \implies P = 5000 \left(1 - \frac{0.07}{4}\right)^{44} = 3769.56\]

4. Kayla borrows $25,000 from Anderson Bank. Anderson Bank charges an annual effective interest rate of \(i\) on the loan. At the end of 7 years, Kayla repays the loan by making a payment of $38,090.06.

   Determine \(i\).

   \[25,000(1 + i)^7 = 38,090.06 \implies (1 + i)^7 = 1.5236024\]

   \[i = (1.5236024)^{\frac{1}{7}} - 1 = 0.06200\]
5. Danny invest 10,000 in a fund. Using the Rule of 72, Danny expects to have 20,000 at the end of 10 years.

Determine the amount that Danny will actually have at the end of 10 years.

Under the Rule of 72, money doubles in \( \frac{72}{i} \) years

\[
10 = \frac{72}{i} \quad \Rightarrow \quad i = 7.2\% \quad \Rightarrow \quad \text{Actual Amount} = 10,000(1.072)^{10} = 20,042.31
\]

6. Tyler invest 4000 at an annual effective interest rate of 5%. Let \( X \) be the estimated number of years that it will take for Tyler’s investment to double. Let \( Y \) be the actual number of years that it will take for Tyler’s investment to double.

Determine \( X - Y \).

\[
X = \frac{72}{5} = 14.4
\]

\[
4000(1.05)^Y = 8000 \quad \Rightarrow \quad (1.05)^Y = 2 \quad \Rightarrow \quad Y = \frac{\ln(2)}{\ln(1.05)} = 14.2067
\]

\[
X - Y = 14.4 - 14.2067 = 0.1933
\]

7. Using the Rule of 72 and a compound interest rate of 6%, it can be estimated that 10 will grow to 40 in \( n \) years.

Under simple interest, assuming a simple interest rate equal to 6%, 10 will grow to 40 in \( t \) years.

Calculate \( t - n \).

\[
10(1 + 0.06t) = 40
\]

\[
t = 50
\]

\[
n = \frac{72}{6} \times 2 = 24
\]

\[
t - n = 50 - 24 = 26
\]
Chapter 2, Section 3

8. Joseph borrows 1000 from Sijie. Joseph repays the loan with a payment of $P$ at the end of 2 years and a payment of $P + 100$ at the end of 4 years.

The annual effective interest rate on the loan is 7%.

Calculate $P$.

\[
1000 = P (1.07)^{-2} + (P + 100)(1.07)^{-4} \\
1000 - 100(1.07)^{-4} = P (1.07)^{-2} + P(1.07)^{-4} \\
923.7105 = 0.87344P + 0.762895P \\
P = 564.50
\]

9. Adam has 10,000 in his bank account today. Five years ago, he deposited 5000 into his account. Additionally, three years ago, Matt deposited 3800 into his account.

Determine the annual effective interest rate that Adam has earned over the five year period.

**Solution:**

For this problem, use the financial calculator’s cash flow functionality.

CF0=5000
CO1=0
F01=1
CO2=3800
FO2=1
CO3=0
FO3=1
CO4=0
FO4=1
CO5= -10,000
IRR, CPT $\Rightarrow$ IRR=3.1272%
10. Ashton deposits 1000 into a bank account on January 1, 2015. Ashton also deposits 300 on October 1, 2015. On December 31, 2015, Ashton has 1450.

Determine the annual effective interest rate earned by Ashton during 2015.

Use the cash flow function on the calculator and treating each period as a month:

CF0 = 1000
C01 = 0
F01 = 8
C02 = 300
F02 = 1
C03 = 0
F03 = 2
C04 = -1450

\[
\text{IRR \rightarrow CPT} = 1.097955462\%
\]

Annual Effective Rate = \((1.01097955462)^{12} - 1 = 0.140010\)

11. Yu loans Ande 500. Ande agrees to repay the loan at an annual effective interest rate of 6% by making a payment of 274.69 at the end of T months and another payment of 274.69 at the end of 2T months.

Determine T in months.

\[
500 = 274.69(1.06)^{-T/12} + 274.69(1+i)^{-2T/12}
\]

Let \(x = (1.06)^{-T/12}\) \(\Rightarrow 500 = 274.69x + 274.69x^2 \Rightarrow 274.69x^2 + 274.69x - 500 = 0\)

\[
x = \frac{-274.69 \pm \sqrt{(274.69)^2 - 4(274.69)(-500)}}{2(274.39)} = 0.938830677 = (1.06)^{-T/12}
\]

\(\therefore (1.06)^{T/12} = 1/0.938830677 = 1.065154798\)

\[
T = \frac{\ln(1.065154798)}{\ln(1.06)} = 1.083255906 \Rightarrow T = 12 * 1.083255906 = 13\text{ months}
\]
12. Danielle borrows 5000. She repays the loan with payments of 3000 at the end of 2 years and 3000 at the end of 4 years.

Determine the annual effective interest rate paid by Danielle.

\[ 5000 = 3000(1 + i)^{-2} + 3000(1 + i)^{-4} \]

Let \( x = (1 + i)^{-2} \) \( \Rightarrow \) \( 5000 = 3000x + 3000x^2 \) \( \Rightarrow \) \( 3000x^2 + 3000x - 5000 = 0 \)

\[ x = \frac{-3000 \pm \sqrt{(3000)^2 - 4(3000)(-5000)}}{2(3000)} = 0.88443731 \]

\( \therefore (1 + i)^{-2} = 1.130662386 \) \( \Rightarrow \) \( i = (1.130662386)^{0.5} - 1 = 0.063326 \)

\( Or \)

\( CFO = 5000; C01 = 0; C02 = -3000; C03 = 0; C04 = -3000 \)

\( [IRR]\ CPT \) \( \Rightarrow \) 6.3326%
13. Emma, Connor, and Isaac enter into a financial arrangement. Emma agrees to pay 50,000 to Connor at time 0. Two years later, at time 2, Emma and Connor each pay 25,000 to Isaac. At the end of the fourth year (time 4), Connor pays 30,000 to Emma and Isaac pays 60,000 to Emma.

Determine Emma’s annual rate of return on under this financial arrangement.

\[ 90000 = 50000(1+i)^4 + 25000(1+i)^2 \]

Let \( x = (1+i)^2 \)

\[ 50000x + 25000 - 90000 = 0 \]

\[ x = \frac{-25000 \pm \sqrt{(-25000)^2 - 4(50000)(-90000)}}{2(50000)} = 1.114734406 \]

\[ 1.114734406 = (1+i)^2 \]

\[ i = (1.114734406)^{0.5} - 1 = 5.5810\% \]
14. Christian, Ethan, and Yifei enter into a financial arrangement. Christian agrees to pay Ethan 3000 today. Christian also agrees to pay Yifei 1000 at the end of one year. At the end of three years, Yifei will pay Christian 4000. At the end of two years, Ethan will pay X to Yifei and 1000 to Christian.

Using the bottom line approach, the annual yield rate or interest rate is the same for Christian and Yifei.

Calculate X.

**Solution:**

<table>
<thead>
<tr>
<th></th>
<th>Christian:</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>-3000</td>
<td>-1000</td>
<td>+1000</td>
<td>+4000</td>
</tr>
<tr>
<td></td>
<td>Erik:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+3000</td>
<td></td>
<td>-X-1000</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Yifei:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>+1000</td>
<td>+X</td>
<td></td>
<td>-4000</td>
</tr>
</tbody>
</table>

For Christian use the financial calculator:

CF0= -3000  
CO1= -1000  
CO2=1000  
CO3=4000  
CPT IRR= 9.14%

Yifei:

\[
0 = 1000(1.0914)^{-1} + X (1.0914)^{-2} - 4000(1.0914)^{-3}
\]

\[
X = \frac{4000(1.0914)^{-3} - 1000(1.0914)^{-1}}{(1.0914)^{-2}}
\]

\[
X = 2573.56
\]
15. Haoran, Jiayi, and Yinzhe are business partners. As part of their partnership, Haoran pays Jiayi 100,000 today. Additionally, at the end of 5 years, Jiayi agrees to pay 120,000 to Yinzhe. Finally, at the end of $T$ years, Yinzhe pays Haoran a total of 172,800.

Using the bottom line approach, all three partners have the same annual yield.

Determine $T$.

Solution:

Using Jiayi’s cash flows, we can find the interest rate using our financial calculators.

$\text{CF0}=100000$
$\text{C01}=0$
$\text{F01}=4$
$\text{C02}=-120000$

$\text{CPT IRR}= 3.7137\%$

Now using this interest rate we can find $T$ either using Haoran’s or Yinzhe’s cash flows. I will use Haoran’s:

$$0 = -100000 + 172800(1.037137)^{-T}$$

$$0.5787 = 1.037137^T$$

$$T = -\frac{\ln(0.5787)}{\ln(1.037137)} = 15\text{ years}$$

Chapter 2, Section 5

16. Sean borrowed 10,000 from Lindsay. Sean will repay the loan with 3 annual payments of 4000. Lindsay reinvests each payment in a fund earning 10% interest.

Determine Lindsay’s yield on this loan taking into account reinvestment.

$$(4000)(1.10)^2 + (4000)(1.10) + 4000 = 13,240$$

$$(10,000)(1+i)^3 = 13,240 \implies (1+i)^3 = 1.324 \implies i = (1.324)^{1/3} - 1 = 0.098068$$
17. Connor lends 8000 to Kurt. Kurt repays the loan with three annual payments of $Q$.

Connor reinvests each payment at an annual effective interest rate of 10%.

Taking into account reinvestment, Connor receives an annual effective return of 8% on the loan.

Determine $Q$.

\[
8,000(1.08)^3 = Q(1.1)^2 + Q(1.1) + Q
\]

\[
10,077.696 = 3.31Q
\]

\[
Q = 3,044.62
\]

18. Courtney loans 5000 to Matt so he can buy a used car. Matt agrees to repay the loan with payments of 2000 at the end of each year for three years.

Courtney reinvests each payment from Matt at an interest rate of $i$.

At the end of three years, Courtney has realized an annual yield of 12%.

Calculate $i$.

\[
5000(1.12)^3 = 2000(1+i)^2 + 2000(1+i) + 2000
\]

\[
0 = 2000(1+i)^2 + 2000(1+i) - 5024.64
\]

Let \(x = (1+i)\)

\[
2000x^2 + 2000 - 5024.64 = 0
\]

\[
x = \frac{-2000 \pm \sqrt{2000^2 - 4(2000)(-5024.64)}}{2(2000)} = 1.16202
\]

\[
i = 1 - 1.16202 = 16.202\%
\]
Chapter 2, Section 6

19. Jessica has an account at Barman Brokerage House. She has made the following deposits and withdrawals over the last two years.

<table>
<thead>
<tr>
<th>Date</th>
<th>Deposits</th>
<th>Withdrawals</th>
<th>Balance Before Cash Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>January 1, 2009</td>
<td>5000</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>March 1, 2009</td>
<td>5000</td>
<td>0</td>
<td>4000</td>
</tr>
<tr>
<td>July 1, 2009</td>
<td>0</td>
<td>4000</td>
<td>10,000</td>
</tr>
<tr>
<td>May 1, 2010</td>
<td>3000</td>
<td>0</td>
<td>9,000</td>
</tr>
<tr>
<td>December 31, 2010</td>
<td>0</td>
<td>0</td>
<td>15,000</td>
</tr>
</tbody>
</table>

a. Estimate Jessica’s dollar weighted annual return using simple interest.

b. Calculate Jessica’s exact annual dollar weighted return using the cash flow functionality in the BA II Plus.

a. 

\[ A + \sum C_t + I = B; A = 5000; B = 15,000 \]

\[ \sum C_t = 5000 - 4000 + 3000 = 4000 \Rightarrow 5000 + 4000 + I = 15,000 \Rightarrow I = 6000 \]

\[ j = \frac{I}{A + \sum C_t(1-t)} = \frac{6000}{5000 + 5000(1 - \frac{1}{12}) - 4000(1 - \frac{1}{4}) + 3000(1 - \frac{2}{3})} \]

\[ = \frac{6000}{7583.33} = 0.7912 \Rightarrow 1 + i = (1.7912)^{1/2} \rightarrow 1 + i = 1.33836 \rightarrow i = 33.836\% \]
b.

Treat Cash Flows as monthly:

- \( CF_0 = 5000 \)
- \( C_{01} = 0; F_{01} = 1 \)
- \( C_{02} = 5000; F_{02} = 1 \)
- \( C_{03} = 0; F_{03} = 3 \)
- \( C_{04} = -4000; F_{04} = 1 \)
- \( C_{05} = 0; F_{05} = 9 \)
- \( C_{06} = 3000; F_{06} = 1 \)
- \( C_{07} = 0; F_{07} = 7 \)
- \( C_{08} = -15,000; F_{08} = 1 \)

\[ \text{IRR} [CPT] = 2.478375127\% \]

Since the periods are months, the IRR is the monthly effective interest rate which is

\[ \frac{i^{(12)}}{12} = 2.478375127\% \]

\[ (1 + i) = \left(1 + \frac{i^{(12)}}{12}\right)^{12} = (1 + 0.02478375127)^{12} \Rightarrow i = 34.149\% \]

Assuming that all cash flows occur on January 1, 2011, Shujing estimates his annual dollar weighted return assuming simple interest to be $4.5383664\%$.

Calculate $X$.

$$A = X \; ; B = 28,400 \; ; k = 0.5$$

$$C = 8000 - 3000 = 5000$$

$$I = B - C - A = 28,500 - 5000 - X = 23,400 - X$$

$$(1 + j)^{0.5} = (1 + i) = 1.045383664 \implies j = 1.045383664^2 - 1 = 0.09287005$$

$$j \approx \frac{I}{A + C(1-k)} = 0.092827005 = \frac{23400 - X}{X + 5000(0.5)}$$

$$0.092827005(X + 2500) = 23,400 - X \implies X = 21,200$$
Chapter 2, Section 7

21. Jessica has an account at Barman Brokerage House. She has made the following deposits and withdrawals over the last two years.

<table>
<thead>
<tr>
<th>Date</th>
<th>Deposits</th>
<th>Withdrawals</th>
<th>Balance Before Cash Flow</th>
</tr>
</thead>
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<tr>
<td>January 1, 2009</td>
<td>5000</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>March 1, 2009</td>
<td>5000</td>
<td>0</td>
<td>4000</td>
</tr>
<tr>
<td>July 1, 2009</td>
<td>0</td>
<td>4000</td>
<td>10,000</td>
</tr>
<tr>
<td>May 1, 2010</td>
<td>3000</td>
<td>0</td>
<td>9,000</td>
</tr>
<tr>
<td>December 31, 2010</td>
<td>0</td>
<td>0</td>
<td>15,000</td>
</tr>
</tbody>
</table>

Calculate Jessica’s annual time weighted return.

\[
1 + j_1 = \frac{4000}{5000} = .8
\]
\[
1 + j_2 = \frac{10,000}{4000 + 5000} = 1.111111
\]
\[
1 + j_3 = \frac{9000}{10,000 - 4000} = 1.5
\]
\[
1 + j_4 = \frac{15,000}{9000 + 3000} = 1.25
\]
\[
j^{TW} = (0.8)(1.1111)(1.5)(1.25) = 1.66667
\]
\[
1 + i^{TW} = (1.66667)^{1/2} \rightarrow i^{TW} = 29.099\%
\]
22. Aakish invested 100,000 in the Tatum Trust Fund.

After 6 months Aakish’s investment had grown to 120,000 and he withdrew 30,000 in order to spend the summer in Europe.

At the end of the summer, he had 10,000 left over that he did not spend in Europe. Aakish decided to deposit that money back into the Tatum Trust Fund. Prior to his deposit, the amount of money that he had in the Fund was 95,000.

Aakish made no further withdrawals or deposits. At the end of two years, Aakish had 99,000.

Calculate the annual effective time weighted return earned by Aakish.

\[
1 + j_1 = \frac{120,000}{100,000} = 1.2
\]
\[
1 + j_2 = \frac{95,000}{90,000} = 1.055556
\]
\[
1 + j_3 = \frac{99,000}{105,000} = 0.942857
\]

\[
1 + j_{rw} = (1.2)(1.055556)(0.942857) = 1.194285714
\]
\[
1 + i_{rw} = (1.194285714)^{0.5}
\]
\[
i_{rw} = 0.092834
\]
23. Brad invests in the Medina Mutual Fund. Over the next two years, Brad realizes an annual time weighted yield of 12.5%.

Brad initially invests 100,000 with Medina and has 103,551.14 at the end of two years. During the two year period, Brad also withdrew an amount of \( C \) to buy a new car. Before the amount was withdrawn, the fund was worth 110,000.

Calculate \( C \).

First, let’s find the time weighted yield for the period of two years.

\[
(1 + j_{TW})^{UT} = (1 + i_{TW})
\]

\[j_{TW} = (1.125)^2 - 1 = 0.265625\]

Now we can solve for \( C \)

\[
1 + 0.265625 = \left( \frac{110,000}{100,000} \right) \left( \frac{103,551.14}{110,000 - c} \right)
\]

\[
c = 110000 - \left( \frac{110,000}{100,000} \right) \left( \frac{103,551.14}{1.265625} \right) = 20,000
\]