Chapter 5, Section 2

1. Xin has a loan for 100,000 which is being repaid with level annual payments for 5 years. The annual effective interest rate on the loan is 8%.

Create an amortization table for this loan.

Solution:

No Solution provided.

2. Taylor has a loan of 8000 to be repaid with 5 annual payments of 2000. Determine:
   a. The amount of interest that Taylor will pay over the life of the loan.
   b. The amount of principal that Taylor will pay over the life of the loan.

Solution:

a. Total interest = Total Payments – Total Principal
   = 5*(2000) – 8000 = 2000
b. Total Principal = amount of loan = 8000

3. Delaney has a loan which is being repaid with level annual payments for 12 years. The principal paid on the loan will be 10,000 and the interest paid on the loan will be 2000.

Calculate the interest rate on the loan.

Solution:

Total payments = Total Interest + Total Principal = 2000 + 10,000 = 12,000
Each payment = 12,000/12 payments = 1000
N= 12, PMT = -1000, PV = 10,000 CPT I/Y = 2.922854%
4. Chenglin has a mortgage loan which is being repaid with level payments at the end of each month for 30 years. The amount borrowed was 400,000 and the interest rate on the loan was 8% compounded monthly.

Calculate the amount of principal repaid in the 135th payment.

Solution:

\[
N = 360, \ I/Y = \frac{7}{12} = 0.66666666 \ , \ PV = -400,000, \ CPT \ PMT = 2935.058
\]

\[
2^{ND} \ Amort \ P1 = 135 \ \ P2 = 135 \ \rightarrow \ PRN = 653.81
\]

\[
Qv^{n-k+1} \rightarrow 2935.058v^{360-135+1} = 653.81
\]

5. Haokun borrowed money to buy a new car. Payments are made monthly. The loan has a nominal rate of interest of 15% compounded monthly. Immediately after the 18th payment, Haokun has an outstanding loan balance of 9500.

Calculate the amount of interest in the 19th payment

Solution:

\[
Interest = (OLB_{k-1})(i)
\]

\[
9500 \left( \frac{0.15}{12} \right) = 118.75
\]
6. Daniel took a loan to buy a new couch for his apartment. He is making monthly payments and the loan has a nominal interest rate of 9% compounded monthly. Immediately after the 8th payment, Daniel still owes 800 on his loan.

The principal in his 9th payment is 90.

Determine the amount of the 9th payment.

**Solution:**

Interest + Principal = Total Payment

Interest = \(800 \left( \frac{0.09}{12} \right) = 6\)

\(6 + 90 = 96\)

7. A loan is being repaid with level monthly payments. The loan has an interest rate of 4.8% compounded monthly. The principal in the 20th payment is 1000.

Calculate the principal in the 10th payment.

**Solution:**

\[1000 \left( 1 + \frac{0.048}{12} \right)^{10-20} = 1000(1.004)^{-10} = 960.87\]
8. Avleen bought a car with a loan which Avleen is repaying with level monthly payments. The principal in the 10th payment is 511.07. The principal in the 15th payment is 542.48.

Calculate the annual effective interest rate on Avleen's loan.

Solution:

\[
511.07 \left(1 + \frac{t^{(12)}}{12}\right)^{15-10} = 542.48
\]

\[
\left(1 + \frac{t^{(12)}}{12}\right)^5 = 1.061459291 \implies \left(1 + \frac{t^{(12)}}{12}\right) = 1.012000364
\]

\[
(1 + i) = \left(1 + \frac{t^{(12)}}{12}\right)^{12} = 1.1538996 \implies i = 1.1538996 - 1 = 15.38996\%
\]

9. A loan is being repaid with level annual payments of 1000. The interest rate on the loan is an annual effective rate of 8%.

The interest in the 5th payment is 768.29.

Calculate the interest in the 10th payment.

Solution:

Principal in 5th: Amount of Payment less the Interest in the Payment = 1000 – 768.29 = 231.71

Principal in 10th: 231.71(1.08)^5 = 340.46 since the principal is a geometric sequence

Interest in 10th: 1000 – 340.46 = 659.54
10. KC has a loan which has an outstanding loan balance of 43,000 immediately after the 12th payment. The next two monthly payments are 1000 each. The interest rate on the loan is a nominal rate of 9% compounded monthly.

Calculate KC’s outstanding loan balance immediately after the 14th payment.

**Solution:**

\[ 43,000 \left( \frac{0.09}{12} \right) = 322.50 \text{ is interest} \rightarrow 1000 - 322.50 = 677.50 \text{ is the principal} \]

\[ 43,000 - 677.50 = 42,322.50 \text{ is the outstanding loan balance} \]

\[ (42,322.50) \left( \frac{0.09}{12} \right) = 317.42 \rightarrow 1000 - 317.42 = 682.58 \]

\[ 42,322.50 - 682.58 = 41,639.92 \]

11. A loan is being repaid with annual payments for 20 years. The principal in the 5th payment is $4236.99. The principal in the 10 payment is $5670.05.

Calculate the amount of the loan to the nearest dollar.

**Solution:**

\[ 4236.99(1 + i)^5 = 5670.05 \rightarrow i = .06 \]

Principal in kth payment = \( Q v^{n-k+1} \rightarrow Q \left( \frac{1}{1.06} \right)^{20-5+1} = 4236.99 \Rightarrow Q = 10,763.44 \]

N=20, I/Y = 6, PMT=-10763.44, CPT PV = 123,455.81
12. A 30 year mortgage is being repaid with level monthly payments. The principal in the 30\textsuperscript{th} payment is 90.43. The principal in the 60\textsuperscript{th} payment is 106.61.

Calculate the interest in the 90\textsuperscript{th} payment.

\textbf{Solution:}

\[ 90.43(1 + i)^{30} = 106.61 \rightarrow i = 0.005501788 \]

Principal in 90\textsuperscript{th} = 106.61(1.005501788)^{30} = 125.68

\[ Qv^{360-30+1} = 90.43 \rightarrow Q = 555.95 \]

Interest = Payment less Principal ==> 555.95 - 125.68 = 430.27

13. A loan is being repaid with 30 annual payments. The payments in odd numbered years (years 1, 3, 5, \ldots, 29) are 10,000. The payments in the even numbered years (years 2, 4, 6, \ldots, 30) are 20,000.

The loan has an annual effective interest rate of 10%.

Calculate the interest in the payment of 10,000 made at the end of the 29\textsuperscript{th} year.

\textbf{Solution:}

Interest in the 29th payment = \((OLB_{28})i)\)

\[ OLB_{28} = PV \text{ of Future Payments } = 10,000v + 20,000v^2 = \]

\[ (10,000)(1.1)^{-1} + (20,000)(1.1)^{-2} = 25,619.83 \]

\[ Interest = (25,619.83)(0.1) = 2561.98 \]
14. A loan is being repaid with annual payments for 20 years. The first payment is 100. The second payment is 200. The payments continue to increase until the 20th payment is 2000.

The loan has an annual effective interest rate of 5.5%.

Calculate the amount of principal in the 11th payment.

Solution:

To solve this problem, we must find the $OLB_{10}$. $OLB_{10}$ is the present value of future payments. Future payments at time 10 are 1100 at time 11, 1200 at time 12, 1300 at time 13, ... to 2000 at time 20. Use the P&Q formula.

$$OLB_{10} = 1100a_{10} + \frac{100}{0.055}(a_{10} - 10v^{10})$$

$$= 1100\left(\frac{1-(1.055)^{-10}}{0.055}\right) + \frac{100}{0.055}\left(\frac{1-(1.055)^{-10}}{0.055} - 10(1.055)^{-10}\right)$$

$$= 1100(7.537625829) + \frac{100}{0.055}(7.537625829 - 5.854305794) = 11,351.97$$

Interest in 11th payment is $(OLB_{10})(i) = (11,351.97)(0.055) = 624.36$

Principal = $Payment - Interest = 1100 - 624.36 = 475.64$
15. A loan of 100,000 is being repaid with geometrically increasing payments at the end of each year for 20 years. The first payment is $Q$. The second payment is $Q(1.05)$. The third payment is $Q(1.05^2)$. The payments continue to increase until $Q(1.05^{19})$ is paid at the end of the 20th year.

The annual effective interest rate on the loan is 5%.

Determine the amount of principal in the 19th payment.

**Solution:**

First we must find $Q$

$100,000 = Q + Q(1.05) + Q(1.05)^2 + \ldots + Q(1.05)^{19}$

\[
\frac{Q}{1.05} + \frac{Q(1.05)}{(1.05)^2} + \ldots + \frac{Q(1.05)^{19}}{(1.05)^{20}} = \frac{Q}{1.05} + \frac{Q}{1.05} + \ldots + \frac{Q}{1.05} = 20 \left( \frac{Q}{1.05} \right)
\]

$Q = \frac{100,000}{20}(1.05) = 5250$

We need $\text{OLB}_{18}$ to find the principal in the 19th payment.

$\text{OLB}_{18} = \text{Present Value of Future Payments} = 5250(1.05)^{18} + 5250(1.05)^{19} v$

$= 5250(1.05)^{17} + 5250(1.05)^{17} = 24,066.19$

$\text{Interest} = (24,066.19)(0.05) = 1203.31$

$\text{Principal} = \text{Payment} - \text{Interest} = 5250(1.05)^{18} = 1203.31 = 11,431.44$
16. Benzinger Corporation has a $200,000 loan that is being repaid with annual payments over the next 8 years using the Sinking Fund method.

The annual effective interest rate on the loan is 5%.

The sinking fund earns an annual effective interest rate of 3.5%. The deposit into the sinking fund is determined so that the sinking fund will exactly repay the loan at the end of 8 years.

a. Determine the amount of interest that Benzinger will pay on the loan each year.

**Solution:**

\[ I = iL = (0.05)(200,000) = 10,000 \]

b. Calculate the amount of the sinking fund deposit.

**Solution:**

\[ D = \frac{L}{s_{0.035}} = \frac{200,000}{(1.035)^8 - 1} = 22,095.33 \]

Or

\[ N = 8 \quad I/Y = 3.5 \quad FV = 200,000 \quad CPT \quad PMT \Rightarrow 22,095.33 \]

c. Determine the amount that will be in the sinking fund at the end of 4 years.

**Solution:**

\[ Balance = Ds_{0.035} = (22,095.33)s_{0.035} = (22,095.33)\left(\frac{(1.035)^4 - 1}{0.035}\right) = 93,130.55 \]

Or

\[ N = 4 \quad I/Y = 3.5 \quad PMT = 22,095.33 \quad CPT \quad FV \Rightarrow 93,130.55 \]
17. Wang Corporation wants to borrow 500,000 to be repaid with annual payments over ten years.

Burnell Bank offers a loan using the sinking fund method. The interest rate on the loan is \( i \) and the sinking fund will earn 5%. Each year, Wang must pay the interest on the loan and make a payment into the sinking fund. The payments into the sinking fund are such that the amount in the sinking fund after 10 years will exactly repay the loan.

Norka Bank offers a loan based on the amortization method and an annual effective interest rate of 6.5%. The amount of the payment under this loan is exactly equal to the sum of the interest payment and sinking fund deposit on the loan from Burnell Bank.

Calculate \( i \), the annual effective interest rate on the loan from Burnell Bank?

**Solution:**

First calculate Norka payment: \( N=10, \ I/Y = 6.5, \ PV = -500,000, \ CPT \ PMT = 69,552.35 \)

\( I + D = 69,552.35 \)

\[
D = \frac{500,000}{s_{10|0.05}}
\]

\( N=10, \ I/Y = 5, \ PMT = -1, \ CPT \ FV = 12.57789 \)

\[
D = \frac{500,000}{s_{10|0.05}} = \frac{500,000}{12.57789} = 39,752.29
\]

\( I = 69,552.35 - 39,752.29 = 29,800.06 \)

\[
I = iL \rightarrow i = \frac{29800.06}{500000} = 0.0596
\]
18. Sarah has a sinking fund loan of 500,000. She is repaying the loan with monthly interest payments using an interest rate of 12% compounded monthly and a monthly sinking fund deposit for the next 10 years. The sinking fund earns an interest rate of 8% compounded monthly. The sinking fund deposits are determined so that the amount in the sinking fund at the end of 10 years will exactly equal the amount of the loan.

Sarah decides to repay the loan balance of 500,000 at the end of 8 years using the amount in the sinking fund plus a lump sum payment of $P$.

Calculate $P$.

Solution:

The amount to be repaid at the end of 8 years is 500,000 since the interest has been paid each month. Therefore, the balance in the sinking fund plus $P$ must equal 500,000. We need to find the balance in the sinking fund after 8 years (96 months). This problem has the additional wrinkle that payments are made monthly.

\[ i^{(12)} = 0.08 \implies \frac{i^{(12)}}{12} = \frac{0.08}{12} = 0.006666667 \]

\[ D = \frac{500,000}{s_{120|0.006666667}} = \frac{500,000}{(1.006666667)^{120} - 1} = 2733.046323 \]

Balance after 8 years = $(2733.046323)(s_{96|0.006666667}) = 365,869.05$

\[ P = 500,000 - 365,869.05 = 134,130.95 \]

Or

\[ N = 120 \quad I/Y = 8/12 \quad FV = 500,000 \quad CPT \quad PMT \implies 2733.046323 \]

\[ N = 96 \quad CPT \quad FV \implies 365,869.05 \]

\[ P = 500,000 - 365,869.05 = 134,130.95 \]
Answers

1. Not Given. The table is self checking as the OLB should be zero at the end.
2.
   a. 2000
   b. 8000
3. 2.9229%
4. 653.81
5. 118.75
6. 96
7. 960.87
8. 15.39%
9. 659.54
10. 41,639.92
11. 123,456
12. 430.26*
13. 2561.98
14. 475.64
15. 11,431.44
16.
   a. 10,000
   b. 22,095.33
   c. 93,130.55
17. 5.96%
18. 134,130.95

*You may get a slightly different answer due to rounding. The same may be true for other questions.