Chapter 6 Homework
Math 373
Spring 2016

Chapter 6, Section 2

1. Anirudha purchases a zero coupon bond for 500. (A zero coupon bond does not pay coupons and only has a maturity value.) The bond will mature in 12 years for 1000. Calculate the annual effective yield rate earned by Anirudha.

Solution:

\[ 500(1+i)^{12} = 1000 \iff (1+i)^{12} = 2 \iff 1+i = 1.059463094 \iff i = 5.9463\% \]

Or

\[ N = 12 \] \[ PV = 500 \] \[ FV = -1000 \] \[ CPT I/Y \] \[ \iff 5.9463\% \]

2. Karolis purchased a 10,000 par value 30 year bond. The bond has a coupon rate of 5% convertible semi-annually and is purchased to yield 6% convertible semi-annually.

Calculate the price that Karolis paid for the bond.

Solution:

Coupon=Fr= (10,000) (0.05/2)=250

N=60, I/Y=6/2=3, PMT=250, FV=10,000, CPT PV=8616.22

Or

\[ P = 250a_{60}^{0.03} + 10,000(1.03)^{-60} = 8616.22 \]
3. Karolis purchased a 20,000 par value 15 year bond. The bond has a coupon rate of 5% convertible semi-annually and is purchased to yield 4% convertible semi-annually.

Calculate the price that Karolis paid for the bond.

Solution:

Coupon=Fr= (20,000) (0.05/2)=500

N=30, I/Y=4/2=2, PMT=500, FV=20,000, CPT PV=22,239.65
Or

\[ P = 500a_{\overline{60}|0.02} + 20,000(1.02)^{-30} = 22,239.65 \]

4. Richardson Corporation issues a 10 year bond. The bond has a par value of 10,000. The bond has a coupon rate of 7% convertible semi-annually. The bond has a maturity value of 12,000.

Calculate the price of this bond to yield 6% convertible semi-annually.

Solution:

Coupon=Fr= (10,000) (0.07/2)=350

N=20, I/Y=6/2=3, PMT=350, FV=12,000, CPT PV=11,851.23
Or

\[ P = 350a_{\overline{20}|0.03} + 12,000(1.03)^{-20} = 11,851.23 \]
5. An $n$ year bond has a par value of 8000. The bond pays semi-annual coupons of 240 and matures for par.

Linfeung purchase the bond for 8715.40 based on a yield of 5% convertible semi-annually.

Determine $n$.

**Solution:**

\[ I/Y = 5/2 = 2.5, \quad PMT = 240, \quad PV = -8715.40, \quad FV = 12,000, \quad CPT N = 24 \rightarrow 12 \text{ years} \]

Or

\[
8715.40 = 240a_{\frac{n}{2}} + 8000(1.025)^{-n} = 240\left(\frac{1-(1.025)^{-n}}{0.025}\right) + 8000(1.025)^{-n} =
\]

\[
\frac{240}{0.025} = \left(\frac{240}{0.025}\right)(1.025)^{-n} + 8000(1.025)^{-n} = 9600 - (9600 - 8000)(1.025)^{-n} = 8715.40
\]

\[
(1.025)^{-n} = \frac{9600 - 8715.40}{1600} \quad \Rightarrow \quad n = -\frac{\ln\left(\frac{9600 - 8715.40}{1600}\right)}{\ln(1.025)} = 24 \Rightarrow 12 \text{ years}
\]
For the next three questions, our standard formula for the price of a bond will not work as the coupons are not level. Therefore, just find the present value of cash flows. The coupons will be an increasing or decreasing annuity and then you must add the present value of the maturity value.

6. A special 30 year bond has annual coupons and a maturity value of 100,000. The annual coupons are 500 in the first year, 1000 in the second year, 1500 in the third year and continue to increase by 500 each year.

Calculate the price of this bond at an annual effective yield rate of 7%.

Solution:

Price = present value of cash flows = present value of coupons + present value of maturity value =

\[
500a_{30\bar{a}} + \left( \frac{500}{0.07} \right) \left( a_{30\bar{a}} - 30v^{30} \right) + 100,000v^{30}
\]

\[a_{30\bar{a}} \Rightarrow \text{N=30, I/Y=7, PMT=-1, CPT PV=12.40904}\]

\[
500(12.40904) + \left( \frac{500}{0.07} \right) (12.40904 \times 3.94101) + 100,000v^{30} = 79,827.14
\]
7. A special 25 year bond has semi-annual coupons and a maturity value of 50,000. The semi-annual coupons are 100 in the first year, 200 in the second year, 300 in the third year and continue to increase in the same pattern.

Calculate the price of this bond at a yield rate of 10% convertible semi-annually.

Solution:

$$\text{Present Value of Coupons} = \left( \text{FirstPayment} \right) \left[ \frac{\ddot{a}_{25|i=0.1025}}{i^{(m)}} \right]$$

$$\left( \frac{i^{(m)}}{m} \right) = 0.05 \rightarrow i \rightarrow (1.05)^2 = 1.1025 - 1 \rightarrow i = 0.1025$$

$$\text{Price} = (100) \left[ \frac{\ddot{a}_{25|i=0.1025} - 25v_{1025}^{25}}{0.05} \right] + 50,000v_{1025}^{25}$$

$$a_{25|i=0.1025} \Rightarrow N=25, I/Y=10.25, \text{PMT}=-1, \text{CPT PV}=8.905329493$$

$$\ddot{a}_{25|i=0.1025} = a_{25|i=0.1025} = 9.818125766$$

$$\left(100\right) \left[ \frac{9.8181 - 2.180093}{0.05} \right] + 50000v_{1025}^{25} = 19,636.25$$
8. A 10 year special bond has annual coupons and a maturity value of 40,000. The annual coupons are 500 in the first year, 500(0.95) in the second year, 500(0.95)^2 in the third year and continue to decrease in the same pattern.

Calculate the price of this bond at an annual effective yield rate of 8%.

**Solution:**

\[
\text{Price} = \text{Present Value of Cash Flows} = 500v + 500(0.95)v^2 + ... + 500(0.95)^9v^{10} + 40,000v^{10}
\]

\[
= \left[ \frac{500(1.08)^{-1} - 500(0.95)^{10}(1.08)^{-11}}{1-(0.95)/(1.08)} \right] + 40,000v^{10} = 2779.49596 + 40,000v^{10} = 21,307.24
\]

9. A 10-year bond has a maturity value of 10,000 and semi-annual coupons of 500. Calculate the price of this bond immediately after the 13th coupon at a yield rate of 4% convertible semi-annually. (Remember that the price of a bond at any point during its lifetime is the present value of future cash flows.)

**Solution:**

Price is equal to the present value of future cash flows.

\[
N=7, \ I/Y=4/2=2, \ PMT=500, \ FV=10,000, \ CPT \ PV=11,941.60
\]

10. A 10 year bond has a par value of 1000 and a maturity value of 1500. The bond has annual coupons of 50. The bond is purchased to yield 4% annually.

Calculate the amount of discount or premium. Be sure to state whether the amount is a discount or a premium.

**Solution:**

\[
\text{Price} \rightarrow N=10, \ I/Y=4, \ PMT=50, \ FV=1500, \ CPT \ PV=1418.89
\]

P<C : Discount = C-P = 1500 – 1418.89 = 81.11
11. A 20 year bond has a par value of \( F \). The bond has a maturity value of \( 1.25F \). The bond pays semi-annual coupons at a rate of 6\% compounded semi-annually.

The bond is bought at a discount of 28.99 when purchased to yield 5\% convertible semi-annually.

Determine \( F \).

**Solution:**

\[
\begin{align*}
r &= \frac{0.06}{2} = 0.03; \quad F = F; \quad C = 1.25F; \quad j &= \frac{0.05}{2} = 0.025; \quad n = (20)(2) = 40 \\
P &= C - 28.99 = 1.25F - 28.99 \quad \text{Since the bond sells at a discount} \\
P &= Fr_a_{40} + Cv_{40} \Rightarrow 1.25F - 28.99 = F(0.03) \left( \frac{1 - (1.025)^{-40}}{0.025} \right) + 1.25F(1.025)^{-40} \\
\text{Solve for } F.
\end{align*}
\]

\[
1.25F - 28.99 = F(0.753083252) + (0.46553828)F \Rightarrow F = \frac{28.99}{1.25 - 1.218621532} = 923.88
\]

12. A 10 year bond has a par value of 1000 and a maturity value of 1500. The bond as semi-annual coupons of 50. The bond is purchased to yield 8\% convertible semi-annually. Calculate the book value immediately after the 5\textsuperscript{th} coupon.

**Solution:**

Book Value is equal to the present value of future cash flows ➔

\[
\begin{align*}
\text{N}=15, \quad I/Y=4, \quad PMT=50, \quad FV=1500, \quad CPT \ PV=1388.82
\end{align*}
\]
13. A 40 year bond with a par value of 100,000 matures at par. The semi-annual coupons are paid at a rate of 8% convertible semi-annually. The bond is bought to yield 7% convertible semi-annually. Calculate the amortization of premium in the coupon paid at the end of the 15th year of this bond.

**Solution:**
Coupon=100,000(.08/2)=4000

Book Value after 29th coupon → N=51, I/Y=7/2=3.5, PMT=4000, FV=100,000 CPT PV=111,814.31

111,814.31(.035)=3913.50 is the interest in the 30th coupon.

4000 – 3913.50 = 86.50 is the principal in the 30th coupon.

14. A five year 5000 par value bond with annual coupons at an annual rate of 6% is purchased to yield 4% annually.

Calculate the amortization table for this bond.

**Solution:**
Price → N=5, I/Y=4, PMT=5000(.06)=300, FV=5000, CPT PV=5445.18

See excel spreadsheet for amortization table.

15. A four year bond matures for 10,000 and has annual coupons of 1000 at the end of the first year, 2000 at the end of the second year, 3000 at the end of the third year, and 4000 at the end of the fourth year.

This bond is purchased to yield 10% annually.

Create an amortization table for this bond.

**Solution:**
Price = 1000v + 2000v^2 + 3000v^3 + 4000v^4 + 10000v^4 = 14378.12

See excel spreadsheet for amortization table.
16. A bond is purchased to yield 9% convertible semi-annually. The bond has semi-annual coupons of 500. The interest in the 3\textsuperscript{rd} coupon is 369. Calculate the interest in the 6\textsuperscript{th} coupon.

\textbf{Solution:}
Principal in 3\textsuperscript{rd} coupon=\text{Coupon – Interest} = 500-369=131

Principal is a geometric sequence so \(131(1.045)^3 = 149.49\) is the principal in the 6\textsuperscript{th} coupon

Interest in 6\textsuperscript{th} coupon=\text{Coupon – Principal} = 500-149.49=350.51

17. A \(n\)-year bond matures for 1000 and has annual coupons of 100. The yield rate on the bond is 8\% annually.

Calculate the book value immediately after the coupon at time \(n-2\).

\textbf{Solution:}
Book value is present value of future cash flows = Present value of coupon at time \(n-1\) + the present value of the coupon at time \(n\) + present value of the maturity value =

\[
100v + 100v^2 + 1000v^2 = 100(1.08)^{-1} + 1100(1.08)^{-2} = 1035.67
\]
18. A 30 year special bond has annual coupons and a maturity value of 100,000. The annual coupons are 500 in the first year, 1000 in the second year, 1500 in the third year and continue to increase by 500 each year.

Calculate the amount of principal in the 29th coupon assuming that the bond was purchased to yield rate of 5%.

Solution:
We need the book value right after the 28th coupon which is the present value of future cash flows = Present value of 29th coupon + the present value of the 30th coupon + present value of the maturity value.

29th coupon = (29)(500)=14,500 and 30th coupon = (30)(500)=15,000

Then present value =
14,500v + 15,000v^2 + 100,000v^2 = 14,500(1.05)^{-1} + 115,000(1.05)^{-2} = 118,117.91

OR

Present Value = 14500a_{2|} + \left( \frac{500}{0.05} \right) (a_{2|} - 2v^2) + 100,000v^2

a_{2|} => N=2, I/Y=5, PMT=-1, CPT PV=1.85941

14,500(1.85941) + \left( \frac{500}{0.05} \right) (1.85941 - 1.814059) + 100,000v^2 = 118,117.91

Coupon = 14,500
Interest = 118,117.91(.05) = 5905.90
Principal = 14,500 – 5905.90 = 8594.10
19. A bond with a par value of 1000 is bought to yield 8% convertible semi-annually. The bond pays semi-annual coupons at a rate of 6% compounded semi-annually. The bond matures for C. If the Book Value of the Bond immediately after the 5th Coupon is 1100, fill in the following table:

<table>
<thead>
<tr>
<th>Time</th>
<th>Coupon</th>
<th>Interest</th>
<th>Principal</th>
<th>Book Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>5th Coupon</td>
<td></td>
<td></td>
<td></td>
<td>1100</td>
</tr>
<tr>
<td>6th Coupon</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Solution:

6th coupon = 1000(.06/2)=30

\[ I_6 = 1100 \left( \frac{0.08}{2} \right) = 44 \]

\[ P_6 = 30 - 44 = -14 \]

\[ B_6 = 1100 - (-14) = 1114 \]

\[ P_5 = -14(1 + r) = -13.46 \text{ since principal is a geometric sequence} \]

5th coupon = 30

\[ I_5 = 30 - (-13.46) = 43.46 \]

20. The book value of a bond immediately after the 13th coupon is 9,266.25. The bond has level annual coupons of 700. The interest in the 10th coupon is 730.36 while the interest in the 20th coupon is 765.54. Calculate the book value immediately after the 14th coupon.

Solution:

\[ P_{10} = 700 - 730.36 = -30.36 \]

\[ P_{20} = 700 - 765.54 = -65.54 \]

\[-30.36(1 + i)^{10} = -65.54 \]

\[(1 + i)^{10} = 2.15876 \rightarrow i = 8\% \]

9266.25(.08) = 741.30 is the interest in the 14th coupon

700 – 741.30 = -41.30 is the principal in the 14th coupon

9266.25 – -41.3 = 9307.55 is the book value after the 14th coupon
21. A 10-year bond has annual coupons of 120 and matures for 2000. The bond is callable at par at the end of the 5th year, the 7th year, and the 9th year.

Calculate the price that you should pay for this bond in order to assure a yield of 7%.

**Solution:**

<table>
<thead>
<tr>
<th>N</th>
<th>PMT</th>
<th>FV</th>
<th>I/Y</th>
<th>PRICE</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>120</td>
<td>2000</td>
<td>7</td>
<td>1918</td>
</tr>
<tr>
<td>7</td>
<td>120</td>
<td>2000</td>
<td>7</td>
<td>1892.21</td>
</tr>
<tr>
<td>9</td>
<td>120</td>
<td>2000</td>
<td>7</td>
<td>1869.7</td>
</tr>
<tr>
<td>10</td>
<td>120</td>
<td>2000</td>
<td>7</td>
<td><strong>1859.53</strong></td>
</tr>
</tbody>
</table>
22. A 20 year bond has a par value of 1000. The bond matures for par and has a coupon rate of 5% convertible semi-annually. The bond can be called at the end of each of the following years for the following call premium:

<table>
<thead>
<tr>
<th>End of Year</th>
<th>Call Premium</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>175</td>
</tr>
<tr>
<td>14</td>
<td>125</td>
</tr>
<tr>
<td>16</td>
<td>75</td>
</tr>
<tr>
<td>18</td>
<td>25</td>
</tr>
</tbody>
</table>

Calculate the price that you should pay to assure a yield of 3% convertible semi-annually.

Solution:

<table>
<thead>
<tr>
<th>N</th>
<th>I/Y</th>
<th>PMT</th>
<th>FV</th>
<th>PRICE</th>
</tr>
</thead>
<tbody>
<tr>
<td>24</td>
<td>1.5</td>
<td>25</td>
<td>1000+175</td>
<td>1322.72</td>
</tr>
<tr>
<td>28</td>
<td>1.5</td>
<td>25</td>
<td>1000+125</td>
<td>1309.65</td>
</tr>
<tr>
<td>32</td>
<td>1.5</td>
<td>25</td>
<td>1000+75</td>
<td>1299.25</td>
</tr>
<tr>
<td>36</td>
<td>1.5</td>
<td>25</td>
<td>1000+25</td>
<td>1291.23</td>
</tr>
<tr>
<td>40</td>
<td>1.5</td>
<td>25</td>
<td>1000</td>
<td>1299.16</td>
</tr>
</tbody>
</table>
23. A 10 year bond with annual coupons of 1000 matures for 10,000. The bond is callable at the end of 6 years for 12,000 or at the end of 8 years for 11,000. Sandy buys this bond for 14,000.

Determine the minimum annual yield rate that Sandy will earn.

Solution:

<p>| | | | | |</p>
<table>
<thead>
<tr>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>PMT</td>
<td>FV</td>
<td>PV</td>
<td>I/Y</td>
</tr>
<tr>
<td>---</td>
<td>-----</td>
<td>----</td>
<td>----</td>
<td>-----</td>
</tr>
<tr>
<td>6</td>
<td>1000</td>
<td>12000</td>
<td>14000</td>
<td>5.045</td>
</tr>
<tr>
<td>8</td>
<td>1000</td>
<td>11000</td>
<td>14000</td>
<td>4.89</td>
</tr>
<tr>
<td>10</td>
<td>1000</td>
<td>10000</td>
<td>14000</td>
<td>4.856</td>
</tr>
</tbody>
</table>

24. Andrea has the choice of purchasing the following two bonds:

a. Bond A is non-callable 10 year bond that matures for 2000 and pays annual coupons of 100.

b. Bond B is a callable 10 year bond that matures for 2400 and pays annual coupons of 50. Bond B may be called at the end of year 6, 7, 8, or 9 with no call premium.

Each bond has a price of 2000.

Which bond should Andrea purchase and why?

Solution:

Yield on Bond A → N=10, PMT=100, PV=-2000, FV=2000, CPT I/Y=5

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>PMT</td>
<td>FV</td>
<td>PV</td>
<td>I/Y</td>
</tr>
<tr>
<td>---</td>
<td>-----</td>
<td>----</td>
<td>----</td>
<td>-----</td>
</tr>
<tr>
<td>6</td>
<td>50</td>
<td>2400</td>
<td>2000</td>
<td>5.41</td>
</tr>
<tr>
<td>7</td>
<td>50</td>
<td>2400</td>
<td>2000</td>
<td>4.96</td>
</tr>
<tr>
<td>8</td>
<td>50</td>
<td>2400</td>
<td>2000</td>
<td>4.62</td>
</tr>
<tr>
<td>9</td>
<td>50</td>
<td>2400</td>
<td>2000</td>
<td>4.36</td>
</tr>
<tr>
<td>10</td>
<td>50</td>
<td>2400</td>
<td>2000</td>
<td>4.15</td>
</tr>
</tbody>
</table>

Choose A because it has a higher yield rate (5% versus 4.15%).
25. A callable bond matures in 10 years for 1000. The bond pays semi-annual coupons of 42.

   The bond may be called at the end of year 6 or year 8. The call value at the end of year 6 is 1085. The call value at the end of year 8 is 1043.

   Andrew purchases this bond at issue to yield 6% convertible semi-annually. The price is $P_1$.

   Two years after issue, Andrew sells the bond for a price of $P_2$ to yield 5% convertible semi-annually.

   Calculate $P_2 - P_1$.

**Solution:**

<table>
<thead>
<tr>
<th>N</th>
<th>I/Y</th>
<th>PMT</th>
<th>FV</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>3</td>
<td>-42</td>
<td>-1085</td>
<td>1179.07</td>
</tr>
<tr>
<td>16</td>
<td>3</td>
<td>-42</td>
<td>-1043</td>
<td>1177.53</td>
</tr>
<tr>
<td>20</td>
<td>3</td>
<td>-42</td>
<td>-1000</td>
<td>1178.53</td>
</tr>
</tbody>
</table>

$P_1 = 1177.53$

At the end of two years, to find the price, you now have a bond that matures in 8 years and can be called at the end of 4 years or 6 years. You calculate the price at each of these points and then choose the lowest.

<table>
<thead>
<tr>
<th>N</th>
<th>I/Y</th>
<th>PMT</th>
<th>FV</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>2.5</td>
<td>-42</td>
<td>-1085</td>
<td>1191.66</td>
</tr>
<tr>
<td>12</td>
<td>2.5</td>
<td>-42</td>
<td>-1043</td>
<td>1206.35</td>
</tr>
<tr>
<td>16</td>
<td>2.5</td>
<td>-42</td>
<td>-1000</td>
<td>1221.94</td>
</tr>
</tbody>
</table>

$P_2 = 1191.66$

$P_2 - P_1 = 1191.66 - 1177.53 = 14.13$