Chapter 8

Use the following spot yield curve for Problems 1-5:

<table>
<thead>
<tr>
<th>Time</th>
<th>Spot Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>½</td>
<td>1.0%</td>
</tr>
<tr>
<td>1</td>
<td>1.5%</td>
</tr>
<tr>
<td>1 ½</td>
<td>2.0%</td>
</tr>
<tr>
<td>2</td>
<td>2.5%</td>
</tr>
<tr>
<td>2 ½</td>
<td>3.0%</td>
</tr>
<tr>
<td>3</td>
<td>4.0%</td>
</tr>
<tr>
<td>3 ½</td>
<td>5.0%</td>
</tr>
<tr>
<td>4</td>
<td>6.0%</td>
</tr>
</tbody>
</table>

1. Calculate the price of a 2 year bond with a par value of 10,000. The bond matures for par and has a coupon rate of 6% convertible semi-annually.

Solution:

\[ \text{Coupons} = (10000) \left( \frac{0.06}{2} \right) = 300 \]

Use spot rates for each period of time.

\[ P = 300(1.01)^{-1} + 300(1.015)^{-1} + 300(1.02)^{-2} + 300(1.025)^{-2} + 10000(1.025)^{-2} = 10688.99 \]

2. Calculate the present value of a four year annuity due with annual payments of 5000.

Solution: Use spot rates for each period of time.

\[ 5000 + 5000(1.015)^{-1} + 5000(1.025)^{-2} + 5000(1.04)^{-3} = 19130.16 \]
3. Two years from today, Rinat will receive 100,000. Three years from today, Rinat will receive 200,000.

How much will Rinat have four years from today?

Solution:

We need to use forward rates which can be found from spot rates.

\[
(1 + s_2)^2 (1 + f_{2,4})^2 = (1 + s_4)^4
\]

\[
(1.025)^2 (1 + f_{2,4})^2 = (1.06)^4
\]

\[
(1 + f_{2,4})^2 = 1.201643745
\]

\[
(1 + s_3)^3 (1 + f_{3,4}) = (1 + s_4)^4
\]

\[
(1.04)^3 (1 + f_{3,4}) = (1.06)^4
\]

\[
(1 + f_{3,4}) = 1.12233742
\]

Now use the forward rates to find the accumulated value at the end of 4 years.

\[
FV = 100000(1 + f_{2,4})^2 + 200000(1 + f_{3,4})
\]

\[
FV = 100000(1.201643745) + 200000(1.12233742) = 344631.86
\]

As an alternative, we can use the spot rates and find the present value at time zero and use this value to find the value at time 4.

\[
PV = 100,000(1.025)^{-2} + 200,000(1.04)^{-3} = 272,980.7114
\]

\[
FV = PV(1.06)^4 = (272,980.7114)(1.06)^4 = 344,631.86
\]
4. Calculate the accumulated value of a 3 year annuity immediate with annual payments of 1000.

Solution:

\[
\left(1 + f_{(1,3)}\right)^2 = \frac{(1 + s_3)^3}{(1 + s_1)} = \frac{1.04^3}{1.015} = 1.10824
\]

First find the forward rates,

\[
(1 + f_{(2,3)}) = \frac{(1 + s_3)^3}{(1 + s_2)^2} = \frac{1.04^3}{1.025^2} = 1.07066
\]

Then find the accumulated value.

\[
AV = 1000 \left(1 + f_{(1,3)}\right)^2 + 1000 \left(1 + f_{(2,3)}\right) + 1000
\]

\[
AV = 1000 \times 1.0824 + 1000 \times 1.07066 + 1000
\]

\[
AV = 3178.90
\]

As an alternative, we can use the spot rates and find the present value at time zero and use this value to find the value at time 3.

\[
PV = 1000(1.015)^{-1} + 1000(1.025)^{-2} + 1000(1.04)^{-3} = 2826.04243
\]

\[
AV = PV(1.04)^3 = (2826.04243)(1.04)^3 = 3178.90
\]
5. A four year bond matures for 5000 and has annual coupons of 125. The price of the bond is calculated using the spot yield curve.

Determine the annual yield rate on the bond.

Solution:

First, let's find the present value using spot rates.

\[
PV = \frac{125}{(1 + s_1)} + \frac{125}{(1 + s_2)^2} + \frac{125}{(1 + s_3)^3} + \frac{5125}{(1 + s_4)^4}
\]

\[
= \left( \frac{125}{1.015} \right) + \left( \frac{125}{1.025} \right)^2 + \left( \frac{125}{1.04} \right)^3 + \left( \frac{5125}{1.06} \right)^4 = 4412.734
\]

Then use your calculator to find the interest rate,

\[
PV = -4412.734 \quad PMT = 125 \quad N = 4 \quad FV = 5000 \quad CPT \quad I/Y = 5.88
\]

6. You are given the following yield curve:

<table>
<thead>
<tr>
<th>Term (t)</th>
<th>Spot Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.060</td>
</tr>
<tr>
<td>2</td>
<td>0.065</td>
</tr>
<tr>
<td>3</td>
<td>0.069</td>
</tr>
<tr>
<td>4</td>
<td>0.072</td>
</tr>
</tbody>
</table>

Calculate the 1 year deferred 3 year forward rate \( f_{(1,4)} \).

Solution:

\[
(1 + f_{(1,4)})^3 = \frac{(1 + s_4)^4}{(1 + s_1)} = \frac{(1.072)^4}{(1.06)}
\]

\[
f_{(1,3)} = \left( \frac{(1.072)^4}{(1.06)} \right)^{1/3} - 1 = 0.07603
\]

\[= 7.603\%\]
7. You are given the following two bonds:

<table>
<thead>
<tr>
<th>Term</th>
<th>Annual Coupon</th>
<th>Maturity Value</th>
<th>Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Year Bond</td>
<td>60</td>
<td>1000</td>
<td>1000.00</td>
</tr>
<tr>
<td>2 Year Bond</td>
<td>80</td>
<td>1000</td>
<td>1055.06</td>
</tr>
</tbody>
</table>

Calculate the 2 year spot interest rate.

Solution:

\[
1000 = \frac{1060}{1 + s_1}
\]

\[
s_1 = \frac{1060}{1000} - 1 = 6\%
\]

\[
1055.06 = \frac{80}{1 + s_1} + \frac{1080}{(1 + s_2)^2}
\]

\[
\frac{1080}{(1 + s_2)^2} = 1055.06 - \frac{80}{1.06} = 979.588
\]

\[
s_2 = \left( \frac{1080}{979.588} \right)^{0.5} - 1 = 5\%
\]

\[
s_2 = 5\%
\]
8. You are given the following spot yield curve:

<table>
<thead>
<tr>
<th>t</th>
<th>s_t</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4.0%</td>
</tr>
<tr>
<td>2</td>
<td>3.5%</td>
</tr>
<tr>
<td>3</td>
<td>s_3</td>
</tr>
<tr>
<td>4</td>
<td>s_4</td>
</tr>
</tbody>
</table>

The present value of a four year annuity due of 100 is 379.05.

The price of a 4 year bond with a maturity value of 2000 and annual coupons of 50 is 1875.12.

Calculate the three year deferred one year forward interest rate.

**Solution:**

First, we will need to use the given information to find the spot rates for year 3 and year 4 and then we can use them to find the forward interest rate.

\[
379.05 = 100 + \left( \frac{100}{1+s_1} \right) + \left( \frac{100}{(1+s_2)^2} \right) + \left( \frac{100}{(1+s_3)^3} \right)
\]

\[
\left( \frac{100}{(1+s_3)^3} \right) = 379.05 - 100 + \left( \frac{100}{1.04} \right) + \left( \frac{100}{(1.035)^2} \right) = 89.545
\]

\[
s_3 = \left( \frac{1000}{89.545} \right)^{1/3} - 1 = 0.037495
\]

\[
1875.12 = \left( \frac{50}{1+s_1} \right) + \left( \frac{50}{(1+s_2)^2} \right) + \left( \frac{50}{(1+s_3)^3} \right) + \left( \frac{2050}{(1+s_4)^4} \right)
\]

\[
\left( \frac{2050}{(1+s_4)^4} \right) = 1875.12 - \left( \frac{50}{1.04} \right) + \left( \frac{50}{(1.035)^2} \right) + \left( \frac{50}{(1.037495)^3} \right) = 1735.595
\]

\[
s_4 = \left( \frac{2050}{1735.595} \right)^{1/4} - 1 = 0.0425
\]

\[
f_{(3,4)} = \left( \frac{(1+s_4)^4}{(1+s_3)^3} \right) - 1 = \left( \frac{(1.0425)^4}{(1.037495)^3} \right) - 1 = 5.77\%
\]
Answers

1. 10,688.99
2. 19,130.16
3. 344,631.86
4. 3178.90
5. 5.88%
6. 7.603%
7. 5.00%
8. 5.77%