There are 4 versions of the Quiz. Find your Version!
1. Kasey invests 10,000 in an account with an accumulation function of $a(t) = 1 + 0.06t$.

   Alan invests $X$ in an account earning compound interest at a rate of $i$.

   During the $10^{th}$ year, Kasey and Alan earn the same effective interest rate.

   At the end of 12 years, Kasey and Alan have the same amount of money in their accounts.

   Determine $X$.

Solution:

Under simple interest, $i_{10} = \frac{a(10) - a(9)}{a(9)} = \frac{s}{1 + s(n-1)} = \frac{0.06}{1 + 0.06(10 - 1)} = \frac{0.06}{1.54} = 0.038961039$

Under compound interest, $i_{10} = i = 0.038961039$

$A(12)^{Kasey} = A(12)^{Alan} \implies (10,000)[1 + 0.06(12)] = X(1.038961039)^{12}$

$\therefore X = \frac{(10,000)[1 + 0.06(12)]}{(1.038961039)^{12}} = 10,872.83$
2. Cora invests 25,000 in a fund that earns a force of interest of $\delta_t = 0.04t + 0.003t^2$ for 5 years. At the end of 5 years, the money in the fund is moved to a bank account that earns a nominal rate of interest of 8% compounded quarterly. Cora leaves the money in the bank account for 2.5 years.

How much will Cora have at the end of 7.5 years?

**Solution:**

\[
\text{Amount} = 25,000 \left[ \int_0^5 (0.04t + 0.003t^2) \, dt \right] \left( 1 + \frac{0.08}{4} \right)^{(4)(2.5)} = \\
(25,000) \left[ \frac{0.04t^2}{2} + \frac{0.003t^3}{3} \right]_0^5 (1.02)^{10} = (25,000) \left[ e^{0.625} \right] (1.02)^{10} = 56,934.53
\]
3. Sanchita invests money in an account earning a nominal interest rate of 12% compounded monthly. Provide all answers to five decimal places.

a. Calculate the monthly effective interest rate earning by Sanchita.

Solution:

Monthly effective interest rate = \( \frac{i^{(12)}}{12} = \frac{0.12}{12} = 0.01000 \)

b. Calculate the annual effective interest rate earned by Sanchita.

Solution:

Annual effective interest rate = \( i \)

\[
(1 + i) = \left(1 + \frac{i^{(12)}}{12}\right)^{12} = \left(1 + \frac{0.12}{12}\right)^{12} \Rightarrow i = (1.01)^{12} - 1 = 0.12683
\]

c. Calculate \( d^{(2)} \) which is equivalent to Sanchita’s interest rate.

Solution:

\[
\left(1 - \frac{d^{(2)}}{2}\right)^{-2} = \left(1 + \frac{i^{(12)}}{12}\right)^{12} = \left(1 + \frac{0.12}{12}\right)^{12}
\]

\[
\left(1 - \frac{d^{(2)}}{2}\right) = \left(1 + \frac{0.12}{12}\right)^{-6} \Rightarrow d^{(2)} = 2 \left[1 - (1.01)^{-6}\right] = 0.11591
\]
1. Cora invests 25,000 in a fund that earns a force of interest of \( \delta_t = 0.02t + 0.003t^2 \) for 5 years. At the end of 5 years, the money in the fund is moved to a bank account that earns a nominal rate of interest of 10% compounded quarterly. Cora leaves the money in the bank account for 2.5 years.

How much will Cora have at the end of 7.5 years?

Solution:

\[
Amount = (25,000) \left[ \int_0^5 (0.02t + 0.003t^2) \, dt \right] \left( 1 + \frac{0.10}{4} \right)^{(4)(2.5)} = \\
(25,000) \left[ e^{\frac{0.02t}{2}} \frac{0.003t^3}{3} \right]_{t=0}^{t=5} (1.025)^{10} = (25,000) \left[ e^{0.375} \right] (1.025)^{10} = 46,562.80
\]
2. Sanchita invests money in an account earning a nominal interest rate of 6% compounded monthly. Provide all answers to five decimal places.

   a. Calculate the monthly effective interest rate earning by Sanchita.

   Solution:

   Monthly effective interest rate \( = \frac{i^{(12)}}{12} = \frac{0.06}{12} = 0.00500 \)

   b. Calculate the annual effective interest rate earned by Sanchita.

   Solution:

   Annual effective interest rate = \( i \)

   \[
   (1 + i) = \left(1 + \frac{i^{(12)}}{12}\right)^{12} = \left(1 + \frac{0.06}{12}\right)^{12} \Rightarrow i = (1.005)^{12} - 1 = 0.06168
   \]

   c. Calculate \( d^{(2)} \) which is equivalent to Sanchita’s interest rate.

   Solution:

   \[
   \left(1 - \frac{d^{(2)}}{2}\right)^{-2} = \left(1 + \frac{i^{(12)}}{12}\right)^{12} = \left(1 + \frac{0.06}{12}\right)^{12} = \left(1 + \frac{0.06}{12}\right)^{12} \\
   \left(1 - \frac{d^{(2)}}{2}\right) = \left(1 + \frac{0.06}{12}\right)^{6} \Rightarrow d^{(2)} = 2\left[1 - (1.005)^{6}\right] = 0.05896
   \]
3. Kasey invests 12,000 in an account with an accumulation function of $a(t) = 1 + 0.07t$.

Alan invests $X$ in an account earning compound interest at a rate of $i$.

During the 10th year, Kasey and Alan earn the same effective interest rate.

At the end of 13 years, Kasey and Alan have the same amount of money in their accounts.

Determine $X$.

**Solution:**

Under simple interest, $i_{10} = \frac{a(10) - a(9)}{a(9)} = \frac{s}{1 + s(n-1)} = \frac{0.07}{1 + 0.07(10-1)} = \frac{0.07}{1.63} = 0.042944785$

Under compound interest, $i_{10} = i = 0.042944785$

$A(13)^{Kasey} = A(13)^{Alan} \implies (12,000)[1 + 0.07(13)] = X(1.042944785)^{13}$

$\therefore X = \frac{(12,000)[1 + 0.047(13)]}{(1.042944785)^{13}} = 13,268.37$
1. Sanchita invests money in an account earning a nominal interest rate of 15% compounded monthly. Provide all answers to five decimal places.

   a. Calculate the monthly effective interest rate earning by Sanchita.

   **Solution:**

   Monthly effective interest rate \( i^{(12)} = \frac{0.15}{12} = 0.01250 \)

   b. Calculate the annual effective interest rate earned by Sanchita.

   **Solution:**

   Annual effective interest rate = \( i \)

   \[ (1 + i) = \left( 1 + \frac{i^{(12)}}{12} \right)^{12} = \left( 1 + \frac{0.15}{12} \right)^{12} \Rightarrow i = (1.0125)^{12} - 1 = 0.16075 \]

   c. Calculate \( d^{(2)} \) which is equivalent to Sanchita’s interest rate.

   **Solution:**

   \[ \left( 1 - \frac{d^{(2)}}{2} \right)^{-2} = \left( 1 + \frac{i^{(12)}}{12} \right)^{12} = \left( 1 + \frac{0.15}{12} \right)^{12} \]

   \[ \left( 1 - \frac{d^{(2)}}{2} \right) = \left( 1 + \frac{0.15}{12} \right)^{-6} \Rightarrow d^{(2)} = 2 \left[ 1 - (1.0125)^{-6} \right] = 0.14365 \]
2. Kasey invests $13,000 in an account with an accumulation function of \( a(t) = 1 + 0.08t \).

Alan invests \( X \) in an account earning compound interest at a rate of \( i \).

During the 10th year, Kasey and Alan earn the same effective interest rate.

At the end of 14 years, Kasey and Alan have the same amount of money in their accounts.

Determine \( X \).

Solution:

Under simple interest, 
\[
i_{10} = \frac{a(10) - a(9)}{a(9)} = \frac{s}{1 + s(n-1)} = \frac{0.08}{1 + 0.08(10-1)} = \frac{0.08}{1.72} = 0.04651162791
\]

Under compound interest, 
\[i_{10} = i = 0.04651162791\]

\[A(14)^{\text{Kasey}} = A(14)^{\text{Alan}} \implies (13,000)[1 + 0.08(14)] = X(1.04651162791)^{14}\]

\[\therefore X = \frac{(13,000)[1 + 0.08(14)]}{(1.04651162791)^{14}} = 14,583.52\]
3. Cora invests 8000 in a fund that earns a force of interest of $\delta_i = 0.06t + 0.003t^2$ for 5 years. At the end of 5 years, the money in the fund is moved to a bank account that earns a nominal rate of interest of 12% compounded quarterly. Cora leaves the money in the bank account for 2.5 years.

How much will Cora have at the end of 7.5 years?

**Solution:**

$$Amount = 8000 \left[ \frac{\int_0^5 (0.06t + 0.003t^2) dt}{e^{\int_0^5 (0.06t + 0.003t^2) dt}} \right] \left( 1 + \frac{0.12}{4} \right)^{(4 \times 2.5)} = $$

$$(8000) \left[ \frac{0.06t^2 + 0.003t^3}{2 + 3} \right]_0^5 (1.03)^{10} = (8000) \left[ e^{0.875} \right] (1.03)^{10} = 25,791.10$$
1. Kasey invests $18,000$ in an account with an accumulation function of $a(t) = 1 + 0.04t$.

Alan invests $X$ in an account earning compound interest at a rate of $i$.

During the 10th year, Kasey and Alan earn the same effective interest rate.

At the end of 15 years, Kasey and Alan have the same amount of money in their accounts.

Determine $X$.

Solution:

Under simple interest, $i_0 = \frac{a(10) - a(9)}{a(9)} = \frac{s}{1 + s(n-1)} = \frac{0.04}{1 + 0.04(10-1)} = \frac{0.04}{1.36} = 0.029411765$

Under compound interest, $i_0 = i = 0.029411765$

$A(15)^{Kasey} = A(15)^{Alan} \implies (18,000)[1 + 0.04(15)] = X(1.029411765)^{15}$

$\therefore X = \frac{(18,000)[1 + 0.04(15)]}{(1.029411765)^{15}} = 18,644.71$
2. Sanchita invests money in an account earning a nominal interest rate of 9% compounded monthly. Provide all answers to five decimal places.

a. Calculate the monthly effective interest rate earning by Sanchita.

Solution:

Monthly effective interest rate = \( \frac{i^{(12)}}{12} = \frac{0.09}{12} = 0.00750 \)

b. Calculate the annual effective interest rate earned by Sanchita.

Solution:

Annual effective interest rate = \( i \)

\[
(1 + i) = \left(1 + \frac{i^{(12)}}{12}\right)^{12} = \left(1 + \frac{0.09}{12}\right)^{12} \rightarrow i = (1.0075)^{12} - 1 = 0.09381
\]

c. Calculate \( d^{(2)} \) which is equivalent to Sanchita's interest rate.

Solution:

\[
\left(1 - \frac{d^{(2)}}{2}\right)^{-2} = \left(1 + \frac{i^{(12)}}{12}\right)^{12} = \left(1 + \frac{0.09}{12}\right)^{12}
\]

\[
\left(1 - \frac{d^{(2)}}{2}\right) = \left(1 + \frac{0.09}{12}\right)^{-6} \rightarrow d^{(2)} = 2\left[1 - (1.0075)^{-6}\right] = 0.08768
\]
3. Cora invests 25,000 in a fund that earns a force of interest of \( \delta_t = 0.02t + 0.006t^2 \) for 5 years. At the end of 5 years, the money in the fund is moved to a bank account that earns a nominal rate of interest of 5% compounded quarterly. Cora leaves the money in the bank account for 3.5 years.

How much will Cora have at the end of 8.5 years?

**Solution:**

\[
Amount = (25,000) \left[ e^{\int_0^5 (0.02t + 0.006t^2) \, dt} \right] \left( 1 + \frac{0.05}{4} \right)^{(4)(3.5)} =
\]

\[
(25,000) \left[ e^{\frac{0.02t^2}{2} + \frac{0.006t^3}{3}} \right] (1.0125)^{14} = (25,000) \left[ e^{0.50} \right] (1.0125)^{14} = 49,047.59
\]