There are 4 versions of this quiz.

Please find your version.
1. Ian is receiving an annuity due with quarterly payments for 18 years.

The payments increase with the first payment being 750, the second payment being 1000, the third payment being 1250, etc. The payments continue to increase in this same pattern.

Using an interest rate of 6% compounded quarterly, calculate the present value of the annuity.

Solution:

Since every payment is increasing, this is the P&Q formula. We also note that it is an annuity due.

\[ i^{(4)} = 0.06 \implies \frac{i^{(4)}}{4} = 0.015; P = 750; Q = 250; n = 18 \cdot 4 = 72 \]

\[
PV = \left[ 750a_{\frac{1}{0.015}} + \frac{250}{0.015} \left( a_{\frac{1}{0.015}} - (1.015)^{-72} \right) \right] (1.015) \quad \text{The (1.015) makes it a due.}
\]

\[
= \left[ 750 \left( \frac{1-(1.015)^{-72}}{0.015} \right) + \frac{250}{0.015} \left( \frac{1-(1.015)^{-72}}{0.015} - (1.015)^{-72} \right) \right] (1.015) = 358,124.43
\]
2. Jovi is receiving a 25 year annuity with payments made continuously at a rate of $1600\$ at time $t$.

Calculate the present value of this annuity at an annual effective interest rate of 5%.

**Solution:**

$i = 0.05 \implies \delta = \ln(1.05)$

$$PV = 1600 \left( \frac{\bar{a}_{25|} - 25\nu^{25}}{\delta} \right) = 1600 \left( \frac{1 - (1.05)^{-25}}{\ln(1.05)} - \frac{25(1.05)^{-25}}{\ln(1.05)} \right) = 231,550.24$$
3. Lindsay wants to save for her retirement in 40 years. She deposits monthly payments into a fund that earns an annual effective rate of 7%. The payments are made at the end of each month. Each monthly payment in the first year is 20. Each monthly payment in the second year is 40. Each monthly payment in the third year is 60. The payments continue to increase in the same pattern for 40 years.

How much will Lindsay have in her retirement fund at the end of 40 years?

**Solution:**

Since the payments stay level during each year, but increase year to year, this is the formula that does not follow the rules. We also note that the problem asks for the value at the end so this is the accumulated value.

We are given that \( i = 0.07 \implies \left( 1+\frac{i^{(12)}}{12} \right) = 1.07 \implies \frac{i^{(12)}}{12} = (1.07)^{1/12} - 1 = 0.005654145 \)

\[
PV = 20 \left( \frac{\bar{a}_{40|0.07} - 40(1.07)^{-30}}{0.005654145} \right) = 20 \left( \frac{1-(1.07)^{-40}}{0.07} \frac{0.07}{(1.07) - 40(1.07)^{-40}} \right) = 41,009.60701 \]

\[
AV = PV(1.07)^{40} = (41,009.60701)(1.07)^{40} = 614,096.63
\]
1. Jovi deposits payments into a fund at the end of each month that earns an annual effective rate of 8%. Each monthly payment in the first year is 85. Each monthly payment in the second year is 170. Each monthly payment in the third year is 255. The payments continue to increase in the same pattern for 20 years.

Determine the amount that Jovi will have in the fund at the end of 20 years.

Solution:

Since the payments stay level during each year, but increase year to year, this is the formula that does not follow the rules. We also note that the problem asks for the value at the end so this is the accumulated value.

We are given that \( i = 0.08 \Rightarrow \left(1 + \frac{i^{(12)}}{12}\right)^{12} = 1.08 \Rightarrow \frac{i^{(12)}}{12} = (1.08)^{1/12} - 1 = 0.00643403\)

\[
P_V = 85 \left( \frac{\bar{a}_{20|0.08} - 20(1.08)^{-20}}{0.00643403} \right) = 85 \left( \frac{1 - (1.08)^{-20}}{0.08} \frac{(1.08) - 20(1.08)^{-20}}{0.00643403} \right) = 83,396.25078
\]

\[
A_V = P_V (1.08)^{20} = (83,396.25078)(1.08)^{20} = 388,706.35
\]
2. A 15 year annuity makes payments continuously at a rate of $560r$ at time $t$.

Calculate the present value of this annuity at an annual effective interest rate of 4%.

Solution:

\[ i = 0.04 \implies \delta = \ln(1.04) \]

\[
PV = 560 \left( \frac{\overline{a}_{[15]} - 15v^{15}}{\delta} \right) = 560 \left( \frac{1 - (1.04)^{-15}}{\ln(1.04)} \right) \frac{\ln(1.04)}{15(1.04)^{-15}} = 42,982.06
\]
3. Chenglin is receiving an annuity due with quarterly payments for 30 years. The payments increase with the first payment being 200, the second payment being 225, the third payment being 250, etc. The payments continue to increase in this same pattern.

Using an interest rate of 10% compounded quarterly, calculate the present value of Chenglin's annuity.

**Solution:**

Since every payment is increasing, this is the P&Q formula. We also note that it is an annuity due.

\[
\frac{i^{(4)}}{4} = 0.10 \implies \frac{i^{(4)}}{4} = 0.025; \ P = 200; \ Q = 25; \ n = 30 \cdot 4 = 120
\]

\[
PV = \left[200a_{\frac{0.10}{0.025}} + \frac{25}{0.025} \left(a_{\frac{0.10}{0.025}} - (1.025)^{-120}\right)\right](1.025) \iff \text{The } (1.025) \text{ makes it a due.}
\]

\[
= \left[200\left(\frac{1-(1.025)^{-120}}{0.025}\right) + \frac{25}{0.025} \left(\frac{1-(1.025)^{-120}}{0.025}\right) - (1.025)^{-120}\right](1.025) = 40,304.52
\]
1. Mengzhi is receiving an annuity due with quarterly payments for 20 years. The payments increase with the first payment being 1000, the second payment being 1300, the third payment being 1600, etc. The payments continue to increase in this same pattern.

Using an interest rate of 8% compounded quarterly, calculate the present value of Mengzhi's annuity.

**Solution:**

Since every payment is increasing, this is the P&Q formula. We also note that it is an annuity due.

\[
i^{(4)} = 0.08 \implies \frac{i^{(4)}}{4} = 0.02; P = 1000; Q = 300; n = 20 \cdot 4 = 80
\]

\[
PV = \left[1000a_{\overline{80}|0.02} + \frac{300}{0.02} \left(a_{\overline{80}|0.02} - (1.02)^{-80}\right)\right](1.02) \Leftarrow \text{The (1.02) makes it a due.}
\]

\[
= \left[1000 \left(\frac{1-(1.02)^{-80}}{0.02}\right) + 300 \left(\frac{1-(1.02)^{-80}}{0.02}\right) - (1.02)^{-80}\right](1.02) = 397,576.15
\]
2. Chenglin wants to save for his retirement in 30 years. He deposits monthly payments into a fund that earns an annual effective rate of 10%. The payments are made at the end of each month. Each monthly payment in the first year is 75. Each monthly payment in the second year is 150. Each monthly payment in the third year is 225. The payments continue to increase in the same pattern for 30 years.

Determine the amount that Chenglin will have in his retirement fund at the end of 30 years.

**Solution:**

Since the payments stay level during each year, but increase year to year, this is the formula that does not follow the rules. We also note that the problem asks for the value at the end so this is the accumulated value.

We are given that \( i = 0.10 \implies \left(1 + \frac{f^{(12)}}{12}\right)^{12} = 1.1 \implies \frac{f^{(12)}}{12} = (1.1)^{1/12} - 1 = 0.00797414 \)

\[
PV = 75\left(\frac{a_{30|0.10} - 30(1.10)^{-30}}{0.00797414}\right) = 75\left(\frac{1-(1.10)^{-30}}{0.10} (1.10) - 30(1.10)^{-30} \right) = 81,360.02109
\]

\[
AV = PV(1.10)^{30} = (81,360.02109)(1.10)^{30} = 1,419,683.74
\]
3. A 25 year annuity makes payments continuously at a rate of 3400t at time t.

Calculate the present value of this annuity at an annual effective interest rate of 6%.

Solution:

\[ i = 0.06 \implies \delta = \ln(1.06) \]

\[
P_V = 3400 \left( \frac{\bar{a}_{25}}{\delta} - 25v^{25} \right) = 3400 \left( \frac{1-(1.06)^{-25}}{\ln(1.06)} - 25(1.06)^{-25} \right) = 428,183.20
\]
Math 373  
Quiz 3  
Spring 2016  
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1. A 20 year annuity makes payments continuously at a rate of 1100e^t at time t.

Calculate the present value of this annuity at an annual effective interest rate of 8%.

**Solution:**

\[ i = 0.08 \implies \delta = \ln(1.08) \]

\[
P_V = 1100 \left( \frac{\bar{a}_{20}}{\delta} - 20v^{20} \right) = 1100 \left( \frac{1 - (1.08)^{-20}}{\ln(1.08)} - 20(1.08)^{-20} \right) = 84,540.95
\]
2. Daniel is receiving an annuity due with quarterly payments for 15 years.

   The payments increase with the first payment being 500, the second payment being 540, the third payment being 580, etc. The payments continue to increase in this same pattern.

   Using an interest rate of 12% compounded quarterly, calculate the present value of Daniel’s annuity.

   **Solution:**

   Since every payment is increasing, this is the P&Q formula. We also note that it is an annuity due.

   \[ i^{(4)} = 0.12 \implies \frac{i^{(4)}}{4} = 0.03; P = 500; Q = 40; n = 15 \cdot 4 = 60 \]

   \[
   PV = \left[ 500a_{\overline{60}|0.03} + \frac{40}{0.03} \left( a_{\overline{60}|0.03} - (1.03)^{-60} \right) \right] (1.03) \quad \text{The (1.03) makes it a due.}
   \]

   \[
   = \left[ 500 \left( \frac{1 - (1.03)^{-60}}{0.03} \right) + \frac{40}{0.03} \left( \frac{1 - (1.03)^{-60}}{0.03} - (1.03)^{-60} \right) \right] (1.03) = 38,274.68
   \]
3. Kasey deposits payments into a fund at the end of each month that earns an annual effective rate of 6%.

Each monthly deposit in the first year is 42. Each monthly deposit in the second year is 84. Each monthly deposit in the third year is 126. The payments continue to increase in the same pattern for 25 years.

Determine the amount that Kasey will have in the fund at the end of 25 years.

**Solution:**

Since the payments stay level during each year, but increase year to year, this is the formula that does not follow the rules. We also note that the problem asks for the value at the end so this is the accumulated value.

We are given that \( i = 0.06 \implies \left(1 + \frac{i^{(12)}}{12}\right)^{12} = 1.06 \implies \frac{i^{(12)}}{12} = (1.06)^{1/12} - 1 = 0.004867551 \)

\[
P V = 42 \left( \frac{\ddot{a}_{250.06} - 25(1.06)^{-25}}{0.004867551} \right) = 42 \left( \frac{1 - (1.06)^{-25}}{0.06} \frac{(1.06) - 25(1.06)^{-25}}{0.004867551} \right) = 66,659.07643
\]

\[
A V = P V (1.06)^{25} = (66,659.07643)(1.06)^{25} = 286,092.14
\]