There are 3 versions of the test.

Please find your version.
1. Courtney borrows 4000 to buy new ski equipment. She will repay the loan with level monthly payments over the next 12 months. The loan has an annual effective interest rate of 8.4%.

Calculate the amount of Courtney's monthly payment.

\[
(1 + \frac{i^{(12)}}{12})^{12} = (1 + 0.084) \rightarrow \frac{i^{(12)}}{12} = 0.006744132
\]

In calculator:

\[ PV = 4000 \quad I/Y=0.6744132 \quad N=12 \]

\[ \therefore \text{PMT}=348.13 \]

Or

\[
Pa^{(12)}_{12} = 4000 \implies P = \frac{4000}{1 - (1.006744132)^{-12}} = 347.13
\]
2. Alan won the lottery. He will receive annual payments at the beginning of each year for 25 years. The first payment will be 250,000. The second payment will be 250,000(1.1). The third payment will be 250,000(1.1)^2. The same pattern will continue with each payment being 110% of the previous payment.

Alan takes each payment and invests it in a fund earning an annual effective interest rate of 6.8%.

Calculate the amount that Alan will have at the end of 25 years.

\[ FV = 250,000(1.068)^{25} + 250,000(1.1)(1.068)^{24} + 250,000(1.1)^2(1.068)^{23} + \ldots + 250,000(1.1)^{24}(1.068) \]

\[ FV = \frac{250,000(1.068)^{25} - 250,000(1.1)^{25}}{1 - \frac{1.1}{1.068}} = 47,186,273.40 \]
3. You are given that \( v(t) = \frac{1}{1 + 0.04t^2} \).

Calculate \( i_{10} - \delta_{10} \). Your answer needs to be accurate to 5 decimal places.

\[
v(t) = \frac{1}{a(t)} \rightarrow a(t) = 1 + 0.04t^2
\]

\[
i_{10} = \frac{a(10) - a(9)}{a(9)} = \frac{1 + 0.04(10)^2 - [1 + 0.04(9)^2]}{1 + 0.04(9)^2} = \frac{5 - 4.24}{4.24} = 0.1792453
\]

\[
\delta_t = \frac{a'(t)}{a(t)} = \frac{0.08t}{1 + 0.04t^2} \implies \delta_{10} = \frac{(0.08)(10)}{1 + 0.04(10)^2} = \frac{0.8}{5} = 0.16
\]

\[
i_{10} - \delta_{10} = 0.1792453 - 0.16 = 0.01925
\]
4. Ben, Sammy, and Ian enter into a financial agreement. Under the agreement, Ben will pay Sammy 1000 today. Additionally, Ben will pay Ian 2000 at the end of 3 years.

Sammy will pay Ian X at the end of 2 years.

Ian will pay Ben 4106.30 at the end of 6 years.

Under this arrangement, Ben, Sammy, and Ian all have the same annual effective yield rate.

Determine X.

Ben: \( 1000(1+i)^6 + 2000(1+i)^3 = 4106.30 \)

\[ CF_0 = -1000 \quad CF_1 = 0 \quad F_1 = 2 \quad CF_2 = -2000 \quad F_2 = 1 \]
\[ CF_3 = 0 \quad F_3 = 2 \quad CF_4 = 4106.30 \quad F_4 = 1 \]
\[ \therefore CPT \text{ IRR}=8\% \]

Or

\[ x = (1 + i)^3 \implies 1000x^2 + 2000x - 4106.30 = 0 \]

\[ x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2000 \pm \sqrt{(2000)^2 - 4(1000)(-4106.30)}}{2(1000)} = 1.259712371 \]

\[ x = (1 + i)^3 = 1.259712371 \implies i = (1.259712371)^{\frac{1}{3}} - 1 = 0.08 \]

\[ 1000(1.08)^2 = X \implies X = 1166.40 \]
5. Shujing borrows money to buy a new car. The loan will be repaid with monthly payments of 1000. The interest rate on the loan is a monthly effective rate of 1.5%. Right after the 12th payment, the outstanding loan balance is 25,878.95.

Determine the original amount of the loan.

\[ OL_{12} = L(1 + i)^{12} - 1000s_{12} \]

\[ \Rightarrow 25,878.95 = L(1.015)^{12} - 1000 \left( \frac{(1.015)^{12} - 1}{0.015} \right) \]

\[ L = \frac{25,878.95 + 1000 \left( \frac{(1.015)^{12} - 1}{0.015} \right)}{(1.015)^{12}} = 32,552.33 \]

Or

\[ OL_{12} = Qa_{\frac{n-k}{12}} = 1000a_{\frac{n-12}{12}} \]

\[ PMT = -1000 \quad I/Y=1.5\% \quad PV=25,878.95 \Rightarrow CPT \ N=32.9999 \approx 33 \]

n-12=33 → n=45

\[ PMT = -1000 \quad I/Y=1.5\% \quad N=45 \Rightarrow CPT \ PV = 32,552.34 \]
6. You are given that $\delta_t = 0.03 + 0.005t$. Hannah invests 1000 today and 2000 at the end of two years in an account earning $\delta_t$.

Determine the amount that Hannah will have at the end of 10 years.

\[
FV = 1,000e^0 + 2,000e^2 \int_{0}^{10} (0.03+0.005t) \, dt
\]

\[
FV = 1,000e^0 \left[ \frac{0.03t+0.005}{2} t^2 \right]_0^{10} + 2,000e^2 \left[ \frac{0.03t+0.005}{2} t^2 \right]_0^{10}
\]

\[
FV = 4,965.40
\]
7. Boyan borrowed 100,000 to be repaid with level monthly payments of 2400 plus a drop payment. The annual effective interest rate on the loan is 9%.

Determine the drop payment.

\[
\left(1 + \frac{i^{(12)}}{12}\right)^{12} = (1 + 0.09) \rightarrow \frac{i^{(12)}}{12} = 0.007207323
\]

In calculator:

PV = -100,000 PMT = 2400 I/Y = 0.7207323

CPT N = 49.72669 \rightarrow N = 49

2ND AMORT P1 \leftarrow 1 P2 \leftarrow 49 \downarrow BAL \rightarrow 1733.27

Drop = (1733.27)(1.007207323) = 1745.76
8. Maggie borrows 25,000 from Jessica. The loan will be repaid with three annual payments of 9619.95.

Jessica reinvests the payments at an annual effective rate of $r$.

After reinvestment, Jessica realizes an annual effective return of 7% on the loan.

Determine $r$.

\[25,000(1 + 0.07)^3 = 9619.95(1 + r)^2 + 9619.95(1 + r) + 9619.95\]

\[9619.95(1 + r)^2 + 9619.95(1 + r) - 21,006.125 = 0\]

\[1 + r = \frac{-9619.95 \pm \sqrt{9619.95^2 - 4(9619.95)(-21,006.125)}}{2(9619.95)}\]

\[\therefore r = 0.06 = 6\%\]
9. Yujie has $100,000 in her brokerage account on January 1, 2013.

On March 1, 2013, she had an account balance of $105,000 prior to withdrawing $40,000.

On July 1, 2014, Yujie deposited $52,000. Prior to that deposit, she had a balance of $70,000.

On December 31, 2014, Yujie had a balance of $130,000 in her account.

Using the simple interest rate approximation, estimate the annual dollar weighted return earned by Yujie.

\[ A + C + I = B \implies 100,000 + (-40,000 + 52,000) + I = 130,000 \]

\[ I = 130,000 - [100,000 + (-40,000 + 52,000)] = 18,000 \]

\[ j = \frac{18,000}{100,000 - 40,000 \left( 1 - \frac{2}{24} \right) + 52,000 \left( 1 - \frac{18}{24} \right)} = 23.5808\% \]

\[ 1 + i = (1 + j)^\frac{1}{T} \implies 1 + i = (1.235808)^{\frac{1}{2}} \implies i = 11.1669\% \]
10. Lewis Industries is considering two investments.

The first investment is the purchase of a perpetuity due with annual payments. The cost of the perpetuity would be 1,000,000 and the annual payment would be 80,000.

The second investment is building a new factory. To build the factory, Lewis would invest 1,000,000 today to build the factory. In return, he would expect to realize the following profits at the end of each of the next four years:

<table>
<thead>
<tr>
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<tbody>
<tr>
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</tr>
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<td>200,000</td>
</tr>
<tr>
<td>3</td>
<td>400,000</td>
</tr>
<tr>
<td>4</td>
<td>X</td>
</tr>
</tbody>
</table>

The annual effective interest rate for the perpetuity is equal to the internal rate of return for the new factory.

Determine X.

\[
1,000,000 = 80,000\bar{a}_{\infty} = 80,000\left(\frac{1+i}{i}\right)
\]

\[
\therefore i = 0.086957
\]

\[
1,000,000 = \frac{100,000}{1+i} + \frac{200,000}{(1+i)^2} + \frac{400,000}{(1+i)^3} + \frac{x}{(1+i)^4} \quad \text{where } i = 0.086957
\]

\[
\therefore x = 596,385.19
\]
11. Mengzhi invests 10,000 in an account earning compound interest. At the end of 10 years, Mengzhi has 20,000.

Cora invests 4000 in an account earning simple interest. At end of 8 years, Cora has 8000.

Let $d_s^{\text{Compound}}$ be the effective discount rate earned by Mengzhi in the 5th year.

Let $i_s^{\text{Simple}}$ be the effective interest rate earned by Cora in the 5th year.

Calculate $i_s^{\text{Simple}} - d_s^{\text{Compound}}$. Your answer must be accurate to five decimal places.

\[
10,000(1 - d_s^{\text{Compound}})^{-10} = 20,000 \implies d_s^{\text{Compound}} = 0.066967
\]

\[
4,000(1 + s \cdot 8) = 8,000 \implies s = 0.125 \implies a(s) = 1 + 0.125t
\]

\[
i_s^{\text{Simple}} = \frac{a(5) - a(4)}{a(4)} = 0.0833333
\]

\[
i_s^{\text{Simple}} - d_s^{\text{Compound}} = 0.0833333 - 0.066967 = 0.01637
\]
12. Penelope takes a loan to buy a car. She will make 60 monthly payments of 350 to repay the loan but the payments are deferred with the first payment being made at the end of 12 months and the last payment being made at the end of the 71st month.

Penelope’s interest rate is 9% compounded monthly.

Determine the amount of Penelope’s loan.

\[ i^{(12)} = 0.09 \rightarrow \frac{i^{(12)}}{12} = 0.0075 \]

\[ PV = v^{11} \cdot 350a_{\overline{60}|} = (1 + i)^{-11} \cdot 350 \left[ 1 - \left( \frac{1}{1+i} \right)^{60} \right] \cdot \frac{1 - \left( \frac{1}{1+i} \right)^{60}}{i} \quad \text{where } i=0.0075 \]

\[ PV = 15,530.29 \]
13. Xin invests 13,000 in an account for nine years. The account earns:

   a. An interest rate equivalent to annual effective discount rate of 6% for the first two years;
   
   b. A force of interest of 0.08 for the next three years; and
   
   c. An interest rate of 4% compounded quarterly for the last four years.

Determine the amount in Xin’s account at the end of nine years.

\[ i^{(4)} = 0.04 \rightarrow \frac{i^{(4)}}{4} = 0.01 \]

\[ FV = 13,000(1 - d)^{-2} (e^{3\delta}) \left( 1 + \frac{i^{(4)}}{4} \right)^{4.4} \]

\[ FV = 13,000(1 - 0.06)^{-2} (e^{(0.08)(4)}) (1 + 0.01)^{16} \]

\[ FV = 21,931.094 \]
1. Xin invests 18,000 in an account for nine years. The account earns:
   a. An interest rate equivalent to annual effective discount rate of 8% for the first four years;
   b. A force of interest of 0.06 for the next two years; and
   c. An interest rate of 3% compounded quarterly for the last three years.

Determine the amount in Xin’s account at the end of nine years.

\[ i^{(4)} = 0.03 \rightarrow \frac{i^{(4)}}{4} = 0.0075 \]

\[
FV = 18,000(1 - d)^{-4}(e^{2\delta})\left(1 + \frac{i^{(4)}}{4}\right)^{3.4}
\]

\[
FV = 18,000(1 - 0.08)^{-4}(e^{0.06(2)}) (1 + 0.0075)^{12}
\]

\[ FV = 30,986.833 \]
2. Penelope takes a loan to buy a car. She will make 60 monthly payments of 300 to repay the loan but the payments are deferred with the first payment being made at the end of 12 months and the last payment being made at the end of the 71st month.

Penelope’s interest rate is 6% compounded monthly.

Determine the amount of Penelope’s loan.

\[ i^{(12)} = 0.06 \rightarrow \frac{i^{(12)}}{12} = 0.005 \]

\[ PV = v^{11} \cdot 300a_{\overline{60}|} = (1+i)^{-11} \cdot 300 \left[ 1 - \left( \frac{1}{1+i} \right)^{60} \right] \frac{1}{i} \quad \text{where } i=0.005 \]

\[ PV = 14,689.26 \]
3. Mengzhi invests $10,000 in an account earning compound interest. At the end of 10 years, Mengzhi has $20,000.

Cora invests $4000 in an account earning simple interest. At end of 8 years, Cora has $8000.

Let $d^\text{Compound}_5$ be the effective discount rate earned by Mengzhi in the 5th year.

Let $i^\text{Simple}_5$ be the effective interest rate earned by Cora in the 5th year.

Calculate $i^\text{Simple}_5 - d^\text{Compound}_5$. Your answer must be accurate to five decimal places.

\[
10,000(1 - d^\text{Compound}_5)^{-10} = 20,000 \implies d^\text{Compound}_5 = 0.066967
\]

\[
4,000(1 + s \cdot 8) = 8,000 \implies s = 0.125 \implies a(s) = 1 + 0.125t
\]

\[
i^\text{Simple}_5 = \frac{a(5) - a(4)}{a(4)} = 0.0833333
\]

\[
i^\text{Simple}_5 - d^\text{Compound}_5 = 0.0833333 - 0.066967 = 0.01637
\]
4. Lewis Industries is considering two investments.

The first investment is the purchase of a perpetuity due with annual payments. The cost of the perpetuity would be 1,000,000 and the annual payment would be 90,000.

The second investment is building a new factory. To build the factory, Lewis would invest 1,000,000 today to build the factory. In return, he would expect to realize the following profits at the end of each of the next four years:

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</tr>
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<td>4</td>
<td>X</td>
</tr>
</tbody>
</table>

The annual effective interest rate for the perpetuity is equal to the internal rate of return for the new factory.

Determine X.

\[
1,000,000 = 90,000 \ddot{a}_{\infty} = 90,000 \left( \frac{1+i}{i} \right)
\]

\[ \therefore i = 0.098901 \]

\[
1,000,000 = \frac{100,000}{1+i} + \frac{200,000}{(1+i)^2} + \frac{400,000}{(1+i)^3} + \frac{x}{(1+i)^4}
\]

where \( i = 0.098901 \)

\[ \therefore x = 644,479.144 \]
5. Yujie has 100,000 in her brokerage account on January 1, 2013.

On March 1, 2013, she had an account balance of 108,000 prior to withdrawing 35,000.

On July 1, 2014, Yujie deposited 45,000. Prior to that deposit, she had a balance of 80,000.

On December 31, 2014, Yujie had a balance of 125,000 in her account.

Using the simple interest rate approximation, estimate the annual dollar weighted return earned by Yujie.

\[ A + C + I = B \implies 100,000 + \{-35,000 + 45,000\} + I = 125,000 \]

\[ I = 125,000 - [100,000 + \{-35,000 + 45,000\}] = 15,000 \]

\[ j = \frac{15,000}{100,000 - 35,000 \left(1 - \frac{2}{24}\right) + 45,000 \left(1 - \frac{18}{24}\right)} = 18.94737\% \]

\[ 1 + i = (1 + j)^{\frac{1}{7}} \implies 1 + i = (1.1894737)^{\frac{1}{2}} \implies i = 9.063\% \]
6. Maggie borrows $25,000 from Jessica. The loan will be repaid with three annual payments of $9261.69.

Jessica reinvests the payments at an annual effective rate of $r$.

After reinvestment, Jessica realizes an annual effective return of 6% on the loan.

Determine $r$.

\[
25,000(1+.06)^3 = 9261.69(1+r)^2 + 9261.69(1+r) + 9261.69
\]

\[
9261.69(1+r)^2 + 9261.69(1+r) - 20,513.71 = 0
\]

\[
1+r = \frac{-9261.69 \pm \sqrt{9261.69^2 - 4(9261.69)(-20,513.71)}}{2(9261.69)}
\]

\[
\therefore r = 0.07 = 7\%
\]
7. Boyan borrowed 100,000 to be repaid with level monthly payments of 1900 plus a drop payment. The annual effective interest rate on the loan is 6%.

Determine the drop payment.

\[
\left(1 + \frac{j^{(12)}}{12}\right)^{12} = (1 + 0.06) \rightarrow \frac{j^{(12)}}{12} = 0.004867551
\]

In calculator:
PV = -100,000  PMT = 1900  I/Y = 0.4867551

\[\therefore N = 60.9516 \rightarrow N = 60\]

\[2ND AMORT\] \[P1 \leftarrow 1\] \[P2 \leftarrow 60\] \[\downarrow BAL \implies 1799.56\]

\[Drop = (1799.56)(1.004867551) = 1,808.32\]
8. You are given that $\delta_t = 0.04 + 0.003t$. Hannah invests 1000 today and 2000 at the end of four years in an account earning $\delta_t$.

Determine the amount that Hannah will have at the end of 10 years.

\[
FV = 1,000e^{\int_0^{10} (0.04+0.003t)\,dt} + 2,000e^{\int_4^{10} (0.04+0.003t)\,dt}
\]

\[
FV = 1,000e^{\left[\frac{0.04t+0.003t^2}{2}\right]_0^{10}} + 2,000e^{\left[\frac{0.04t+0.003t^2}{2}\right]_4^{10}}
\]

\[
FV = 4,617.164
\]
9. Shujing borrows money to buy a new car. The loan will be repaid with monthly payments of 1000. The interest rate on the loan is a monthly effective rate of 1.2%. Right after the 12\textsuperscript{th} payment, the outstanding loan balance is 22,946.13.

Determine the original amount of the loan.

$$OLB_{12} = L(1 + i)^{12} - 1000s_{12}$$

$$\implies 22,946.13 = L(1.012)^{12} - 1000 \left( \frac{(1.012)^{12} - 1}{0.012} \right)$$

$$L = \frac{22,946.13 + 1000 \left( \frac{(1.012)^{12} - 1}{0.012} \right)}{(1.012)^{12}} = 30,999.96$$

Or

$$OLB_{12} = Qa_{n-k} = 1000a_{n-12}$$

$$PMT = -1000 \quad I/Y=1.2\% \quad PV=22,946.13 \implies CPT \quad N=26.9999 \approx 27$$

$$n-12=27 \implies n=39$$

$$PMT = -1000 \quad I/Y=1.2\% \quad N=39 \implies CPT \quad PV = 30,999.96$$
10. Ben, Sammy, and Ian enter into a financial agreement. Under the agreement, Ben will pay Sammy $2000 today. Additionally, Ben will pay Ian $1000 at the end of 3 years.

Sammy will pay Ian X at the end of 2 years.

Ian will pay Ben $4226.50 at the end of 6 years.

Under this arrangement, Ben, Sammy, and Ian all have the same annual effective yield rate.

Determine X.

Ben: $2000(1+i)^6 + 1000(1+i)^3 = 4226.50$

$CF_0 = -2000 \quad CF_1 = 0 \quad F_1 = 2 \quad CF_2 = -1000 \quad F_2 = 1$

$CF_3 = 0 \quad F_3 = 2 \quad CF_4 = 4226.50 \quad F_4 = 1$

$\therefore CPT\ IRR=7\%$

Or

$x = (1 + i)^3 \implies 2000x^2 + 1000x - 4226.50 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1000 \pm \sqrt{(1000)^2 - 4(2000)(-4226.50)}}{2(2000)} = 1.225042372$

$x = 1 + i = 1.225042372 \implies i = (1.225042372)^{1/3} - 1 = 0.07$

$2000(1.07)^2 = X \implies X = 2289.80$
11. You are given that \( v(t) = \frac{1}{1 + 0.04t^2} \).

Calculate \( i_{10} - \delta_{10} \). Your answer needs to be accurate to 5 decimal places.

\[ v(t) = \frac{1}{a(t)} \rightarrow a(t) = 1 + 0.04t^2 \]

\[ i_{10} = \frac{a(10) - a(9)}{a(9)} = \frac{1 + 0.04(10)^2 - [1 + 0.04(9)^2]}{1 + 0.04(9)^2} = \frac{5 - 4.24}{4.24} = 0.1792453 \]

\[ \delta_t = \frac{a'(t)}{a(t)} = \frac{0.08t}{1 + 0.04t^2} \implies \delta_{10} = \frac{(0.08)(10)}{1 + 0.04(10)^2} = \frac{0.8}{5} = 0.16 \]

\[ i_{10} - \delta_{10} = 0.1792453 - 0.16 = 0.01925 \]
12. Alan won the lottery. He will receive annual payments at the beginning of each year for 23 years. The first payment will be 200,000. The second payment will be 200,000(1.1). The third payment will be 200,000(1.1)^2. The same pattern will continue with each payment being 110% of the previous payment.

Alan takes each payment and invests it in a fund earning an annual effective interest rate of 6.4%.

Calculate the amount that Alan will have at the end of 23 years.

\[ FV = 200,000(1.064)^{23} + 200,000(1.1)(1.064)^{22} + 200,000(1.1)^2(1.064)^{21} + \ldots + 200,000(1.1)^{22}(1.064) \]

\[ FV = \frac{200,000(1.064)^{23} - 200,000(1.1)^{23}}{1 - \frac{1.1}{1.064}} = 28,307,692.97 \]
13. Courtney borrows 5000 to buy new ski equipment. She will repay the loan with level monthly payments over the next 12 months. The loan has an annual effective interest rate of 9.6%.

Calculate the amount of Courtney's monthly payment.

\[
(1 + \frac{i^{(12)}}{12})^{12} = (1 + 0.096) \rightarrow \frac{i^{(12)}}{12} = 0.007668183
\]

In calculator:

\[PV = 5000 \ I/Y=0.7668183 \ N=12\]

\[\therefore \ PMT=437.73\]

Or

\[Pa_{12} = 5000 \Rightarrow P = \frac{5000}{1 - (1.007668183)^{-12}} = 437.73\]
1. Boyan borrowed 100,000 to be repaid with level monthly payments of 2300 plus a drop payment. The annual effective interest rate on the loan is 7.5%.

Determine the drop payment.

\[
\left(1 + \frac{i^{(12)}}{12}\right)^{12} = (1 + 0.075) \rightarrow \frac{i^{(12)}}{12} = 0.006044919
\]

In calculator:

\[PV = -100,000 \quad PMT=2300 \quad I/Y=0.6044919\]

\[\therefore N=50.5958 \rightarrow N=50\]

\[2 \text{ND} AMORT P1 \leftarrow 1 \quad P2 \leftarrow 50 \downarrow BAL \implies 1363.71\]

\[Drop = (1763.71)(0.006044919) = 1371.95\]
2. Maggie borrows 25,000 from Jessica. The loan will be repaid with three annual payments of 9748.21.

Jessica reinvests the payments at an annual effective rate of \( r \).

After reinvestment, Jessica realizes an annual effective return of 8\% on the loan.

Determine \( r \).

\[
25,000(1 + 0.08)^3 = 9748.21(1 + r)^2 + 9748.21(1 + r) + 9748.21
\]

\[
9748.21(1 + r)^2 + 9748.21(1 + r) - 21,744.59 = 0
\]

\[
1 + r = \frac{-9748.21 \pm \sqrt{9748.21^2 - 4(9748.21)(-21,744.59)}}{2(9748.21)}
\]

\[
\therefore r = 0.075 = 7.5\%
\]
3. Yujie has 100,000 in her brokerage account on January 1, 2013.

On March 1, 2013, she had an account balance of 107,000 prior to withdrawing 37,000.

On July 1, 2014, Yujie deposited 47,000. Prior to that deposit, she had a balance of 75,000.

On December 31, 2014, Yujie had a balance of 125,000 in her account.

Using the simple interest rate approximation, estimate the annual dollar weighted return earned by Yujie.

\[ A + C + I = B \implies 100,000 + \{-37,000 + 47,000\} + I = 125,000 \]

\[ I = 125,000 - [100,000 + \{-37,000 + 47,000\}] = 15,000 \]

\[ j = \frac{15,000}{100,000 - 37,000 \left(1 - \frac{2}{24}\right) + 47,000 \left(1 - \frac{18}{24}\right)} = 19.2719\% \]

\[ 1 + i = (1 + j)^{\frac{1}{T}} \implies 1 + i = (1.192719)^{\frac{1}{2}} \implies i = 9.2117\% \]
4. Lewis Industries is considering two investments.

The first investment is the purchase of a perpetuity due with annual payments. The cost of the perpetuity would be 1,000,000 and the annual payment would be 85,000.

The second investment is building a new factory. To build the factory, Lewis would invest 1,000,000 today to build the factory. In return, he would expect to realize the following profits at the end of each of the next four years:

<table>
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<td>3</td>
<td>400,000</td>
</tr>
<tr>
<td>4</td>
<td>X</td>
</tr>
</tbody>
</table>

The annual effective interest rate for the perpetuity is equal to the internal rate of return for the new factory.

Determine X.

\[
1,000,000 = 85,000 \ddot{a}_{\infty} = 85,000 \left( \frac{1+i}{i} \right)
\]

\[
 \therefore i = 0.092596
\]

\[
1,000,000 = \frac{100,000}{1+i} + \frac{200,000}{(1+i)^2} + \frac{400,000}{(1+i)^3} + \frac{x}{(1+i)^4} \text{ where } i = 0.092596
\]

\[
 \therefore x = 620,061.38
\]
5. Mengzhi invests 10,000 in an account earning compound interest. At the end of 10 years, Mengzhi has 20,000.

Cora invests 4000 in an account earning simple interest. At end of 8 years, Cora has 8000.

Let $d_{5}^{\text{Compound}}$ be the effective discount rate earned by Mengzhi in the 5th year.

Let $i_{5}^{\text{Simple}}$ be the effective interest rate earned by Cora in the 5th year.

Calculate $i_{5}^{\text{Simple}} - d_{5}^{\text{Compound}}$. Your answer must be accurate to five decimal places.

$$10,000(1 - d_{5}^{\text{Compound}})^{-10} = 20,000 \implies d_{5}^{\text{Compound}} = 0.066967$$

$$4,000(1 + s \cdot 8) = 8,000 \implies s = 0.125 \implies a(s) = 1 + 0.125t$$

$$i_{5}^{\text{Simple}} = \frac{a(5) - a(4)}{a(4)} = 0.083333$$

$$i_{5}^{\text{Simple}} - d_{5}^{\text{Compound}} = 0.083333 - 0.066967 = 0.01637$$
6. Penelope takes a loan to buy a car. She will make 60 monthly payments of 325 to repay the loan but the payments are deferred with the first payment being made at the end of 12 months and the last payment being made at the end of the 71st month.

Penelope’s interest rate is 12% compounded monthly.

Determine the amount of Penelope’s loan.

\[ i^{(12)} = 0.12 \rightarrow \frac{i^{(12)}}{12} = 0.01 \]

\[ PV = v^{11} \cdot 325 a_{\overline{60}}^{(1)} = (1+i)^{-11} \cdot 325 \left[ 1 - \frac{\left( \frac{1}{1+i} \right)^{60}}{i} \right] \]

where \( i = 0.01 \)

\[ PV = 13,095.64 \]
7. Xin invests 23,000 in an account for nine years. The account earns:

   a. An interest rate equivalent to annual effective discount rate of 9% for the first three years;

   b. A force of interest of 0.07 for the next four years; and

   c. An interest rate of 2% compounded quarterly for the last two years.

Determine the amount in Xin’s account at the end of nine years.

\[
i^{(4)} = 0.02 \rightarrow \frac{i^{(4)}}{4} = 0.005
\]

\[
FV = 23,000(1 - d)^{-3}(e^{4\delta})\left(1 + \frac{i^{(4)}}{4}\right)^{2.4}
\]

\[
FV = 23,000(1 - 0.09)^{-3}(e^{(0.07)(4)}) (1 + 0.005)^8
\]

\[
FV = 42,027.60
\]
8. Shujing borrows money to buy a new car. The loan will be repaid with monthly payments of 1000. The interest rate on the loan is a monthly effective rate of 1.4%. Right after the 12\textsuperscript{th} payment, the outstanding loan balance is 31,034.67.

Determine the original amount of the loan.

\[ OLB_{12} = L(1 + i)^{12} - 1000s_{12|} \]

\[ \Rightarrow 31,034.67 = L(1.014)^{12} - 1000 \left( \frac{(1.014)^{12} - 1}{0.014} \right) \]

\[ 31,034.67 + 1000 \frac{(1.014)^{12} - 1}{0.014} \]

\[ L = \frac{(1.014)^{12}}{(1.014)^{12}} = 37,241.62 \]

Or

\[ OLB_{12} = Qa_{n-k|} = 1000a_{n-12|} \]

\[ PMT = -1000 \quad I/Y=1.4\% \quad PV=31,034.67 \Rightarrow CPT \ N=41 \]

\[ n-12=41 \quad \rightarrow \quad n=53 \]

\[ PMT = -1000 \quad I/Y=1.2\% \quad N=53 \Rightarrow CPT \ PV = 37,241.62 \]
9. You are given that $\delta_t = 0.05 + 0.003t$. Hannah invests 1000 today and 2000 at the end of six years in an account earning $\delta_t$.

Determine the amount that Hannah will have at the end of 12 years.

\[
FV = 1,000e^{\int_0^6 (0.05 + 0.003t)\,dt} + 2,000e^{\int_6^{12} (0.05 + 0.003t)\,dt}
\]

\[
FV = 1,000e^{\frac{[0.05t + 0.003t^2]}{2}|_0^6} + 2,000e^{\frac{[0.05t + 0.003t^2]}{2}|_6^{12}}
\]

\[
FV = 5,435.93
\]
10. Ben, Sammy, and Ian enter into a financial agreement. Under the agreement, Ben will pay Sammy 1800 today. Additionally, Ben will pay Ian 1500 at the end of 3 years.

Sammy will pay Ian X at the end of 2 years.

Ian will pay Ben 4148.61 at the end of 6 years.

Under this arrangement, Ben, Sammy, and Ian all have the same annual effective yield rate.

Determine X.

Ben: \[1800(1+i)^6 + 1500(1+i)^3 = 4148.61\]

\[CF_0 = -1800 \quad CF_1 = 0 \quad F_1 = 2 \quad CF_2 = -1500 \quad F_2 = 1\]

\[CF_3 = 0 \quad F_3 = 2 \quad CF_4 = 4148.61 \quad F_4 = 1\]

\[\therefore CPT \ IRR=5\%\]

Or

\[x = (1+i)^3 \implies 1800x^2 + 1500x - 4148.61 = 0\]

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1500 \pm \sqrt{1500^2 - 4(1800)(-4148.61)}}{2(1800)} = 1.15762506\]

\[x = (1 + i)^3 = 1.15762506 \implies i = (1.15762506)^{1/3} - 1 = 0.05\]

\[1800(1.05)^2 = X \quad \rightarrow \quad X = 1984.50\]
11. Alan won the lottery. He will receive annual payments at the beginning of each year for 30 years. The first payment will be $100,000. The second payment will be $100,000(1.1). The third payment will be $100,000(1.1)^2$. The same pattern will continue with each payment being 110% of the previous payment.

Alan takes each payment and invests it in a fund earning an annual effective interest rate of 7.2%.

Calculate the amount that Alan will have at the end of 30 years.

\[
FV = 100,000(1.072)^{30} + 100,000(1.1)(1.072)^{29} + 100,000(1.1)^2(1.072)^{28} + \ldots + 100,000(1.1)^{29}(1.072)
\]

\[
FV = \frac{100,000(1.072)^{30} - 100,000(1.1)^{30}}{1 - \frac{1.1}{1.072}} = 35,982,899.10
\]
12. You are given that \( v(t) = \frac{1}{1 + 0.04t^2} \).

Calculate \( i_{10} - \delta_{10} \). Your answer needs to be accurate to 5 decimal places.

\[
v(t) = \frac{1}{a(t)} \rightarrow a(t) = 1 + 0.04t^2
\]

\[
i_{10} = \frac{a(10) - a(9)}{a(9)} = \frac{1 + 0.04(10)^2 - [1 + 0.04(9)^2]}{1 + 0.04(9)^2} = \frac{5 - 4.24}{4.24} = 0.1792453
\]

\[
\delta_{1} = \frac{a'(t)}{a(t)} = \frac{0.08t}{1 + 0.04t^2} \implies \delta_{10} = \frac{(0.08)(10)}{1 + 0.04(10)^2} = \frac{0.8}{5} = 0.16
\]

\[
i_{10} - \delta_{10} = 0.1792453 - 0.16 = 0.01925
\]
13. Courtney borrows 4500 to buy new ski equipment. She will repay the loan with level monthly payments over the next 12 months. The loan has an annual effective interest rate of 10.8%.

Calculate the amount of Courtney's monthly payment.

\[
(1 + \frac{i^{(12)}}{12})^{12} = (1 + 0.108) \Rightarrow \frac{i^{(12)}}{12} = 0.008583007
\]

In calculator:

\[PV = 4500 \quad I/Y = 0.8583007 \quad N = 12\]

\[\therefore \text{PMT} = 396.25\]

Or

\[Pa_{[12]} = 4500 \implies P = \frac{4500}{1 - (1.08583007)^{-12}} = 396.25\]