1. Datsenka Dog Insurance Company has developed the following mortality table for dogs:

<table>
<thead>
<tr>
<th>Age</th>
<th>( l_x )</th>
<th>Age</th>
<th>( l_x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2000</td>
<td>5</td>
<td>1200</td>
</tr>
<tr>
<td>1</td>
<td>1950</td>
<td>6</td>
<td>1000</td>
</tr>
<tr>
<td>2</td>
<td>1850</td>
<td>7</td>
<td>700</td>
</tr>
<tr>
<td>3</td>
<td>1600</td>
<td>8</td>
<td>300</td>
</tr>
<tr>
<td>4</td>
<td>1400</td>
<td>9</td>
<td>0</td>
</tr>
</tbody>
</table>

Datsenka sells an whole life annuity based on the life of a dog who is age 4. The annuity pays 1000 at the beginning of each year as long as the dog is alive.

The interest rate used to calculate all values is 8%.

Calculate the Actuarial Present Value of this annuity.

**Solution:**

\[
APV = 1000\overline{a}_4
\]

\[
1400 \cdot \overline{a}_4 = 1400 + 1200v + 1000v^2 + 700v^3 + 300v^4
\]

\[
\overline{a}_4 = 2.960458183
\]

\[
APV = 1000(2.960458183) = 2960.46
\]
2. The Ismail Insurance Company sells annuities. Ismail sells a 20 year certain and life annuity to Hayqal who is (40). The annuity provides guaranteed payments of 30,000 at the beginning of each year for 20 years. Thereafter payments continue as long as Hayqal is alive.

You are given:

a. Hayqal’s mortality follows the Illustrative Life Table.

b. \( i = 6\% \).

Calculate the Actuarial Present Value of Hayqal’s annuity.

Solution:

\[
APV = 30,000\overline{a}_{20} = 30,000(\overline{a}_{20} + 20\cdot \bar{a}_{40}) = 30,000\left(\frac{1-v^{20}}{d} + 20 \cdot E_{40} \cdot \bar{a}_{60}\right) =
\]

\[
30,000[12.15811649 + (0.27414)(11.1454)] = (30,000)(15.21351645) = 456,405.49
\]
3. The Liang Life Insurance Company has sold whole life insurance policies to 1000 independent lives who are all age 45. The whole life insurance policies provide a death benefit of 5000 at the end of the year of death.

You are given:

a. Mortality follows the Illustrative Life Table.

b. \( i = 6\% \).

The company’s actuary, Ying, wants to set aside funds now so that she is 95% certain that the present value of the benefits actually paid will be less than the funds set aside.

Using the normal distribution, determine the amount that Ying should set aside now.

Solution:

\[
E[Y] = 5000 \cdot A_{0.5} = (5000)(0.2120) = 1006.00
\]

\[
E[Port] = (1000)(1006) = 1,006,000
\]

\[
Var[Y] = 5000^2 \{\bar{A}_{0.5} - (A_{0.5})^2\} = 5000^2[0.06802 - (0.2120)^2] = 688,464
\]

\[
Var[Port] = (1000)(688,464) = 688,464,000
\]

\[
Amount = 1,006,000 + 1.645\sqrt{688,464,000} = 1,049,162.49
\]
4. \( Y \) is the present value random variable for a continuous whole life annuity to (90) which is payable at an annual rate of 1000 per year.

You are given:

a. \( i = 6\% \).

b. \( \delta_p = 1 - \frac{r}{100} \) for \( 0 \leq t \leq 10 \).

Calculate the \( \Pr(Y < 4500) \).

Solution:

\[
Y = 1000 \frac{1 - v^t}{\delta}
\]

\[
\Pr(Y < 4500) = \Pr \left( 1000 \frac{1 - v^t}{\delta} < 4500 \right) = \Pr \left( 1 - \frac{1 - v^t}{\ln(1.06)} < 4.5 \right) = \Pr \left( 1 - v^t < 0.262210087 \right) = \]

\[
\Pr(v^t > 0.737789913) = \Pr(T_v [\ln(1.06^{-1})] > \ln(0.737789913)) = \]

\[
\Pr(T_v < 5.218840998) = 5.218840998 p_{90} = 1 - 5.218840998 p_{90} = \]

\[
1 - \left( 1 - \frac{(5.218840998)^2}{100} \right) = 0.272363014
\]
5. Emily (25) purchases a special whole life insurance policy with increasing death benefits. The death benefit is 50,000 for the first 10 years. It doubles at the end of 10 years to a death benefit of 100,000 if Emily dies between the end of the 10th year and the end of the 20th year. Finally, it doubles again at the end of the 20th year so the death benefit is 200,000 if Emily dies after the end of the 20th year.

The death benefit is payable at the end of the year of death.

The policy has annual net benefit premiums that are non-level and payable for life. The net benefit premiums is $P$ for the first 20 years and then is $0.5P$ thereafter.

You are given:
   a. Mortality follows the Illustrative Life Table.
   b. $i = 6\%$.

Calculate $P$ using the equivalence principle.

Solution:

\[
PVP = PVB
\]

\[
PVB = 50,000 \cdot A_{25} + 50,000 \cdot A_{25} + 100,000 \cdot A_{25} =
\]

\[
50,000 \cdot A_{25} + 50,000 \cdot E_{25} \cdot A_{25} + 100,000 \cdot E_{25} \cdot A_{45} =
\]

\[(50,000)(0.08165) + (50,000)(0.54997)(0.12872) + (100,000)(0.29873)(0.20120) = 13,632.55452
\]

\[
PVP = 0.5P\bar{a}_{25} + 0.5P\bar{a}_{45} = 0.5P(\bar{a}_{25} + \bar{a}_{25} - E_{25} \cdot \bar{a}_{45}) =
\]

\[
0.5P[16.2242 + 16.2242 - (0.29873)(14.1121)] = 14.11634618P
\]

\[
P = \frac{13,632.55452}{14.11634618} = 965.73
\]
6. Chao (30) buys a 40-year endowment insurance with a death benefit of 40,000 from Li Life Insurance Company. The death benefit is payable at the end of the year of death.

You are given:
   a. Mortality follows the Illustrative Life Table.
   b. \( i = 6\% \).
   c. Deaths are uniformly distributed between integral ages.
   d. The net annual benefit premium for this policy is 354.33.

A. Calculate the variance of the loss random variable \( L_n^0 \) for this policy with annual premiums.

B. If the death benefit on the above policy was payable at the moment of death and the premiums were paid monthly, calculate what the monthly net benefit premium would be under the equivalence principle.

Solution:

Part A

\[
Var[L_n^0] = \left[ S + \frac{P}{d} \right] \left[ 2A_{30:40} - (A_{30:40})^2 \right]
\]

\[
A_{30:40} = A_{30} - E_{30} \cdot A_{40} + E_{90} = 0.10248 - (0.29374)(0.23047)(0.51495) + (0.29374)(0.23047) = 0.13531704
\]

\[
2A_{30:40} = 2A_{90} - v_{40} A_{90} + v_{90} A_{40}
\]

\[
= 0.02531 - (1.06)^{-40}(0.29374)(0.23047)(0.30642) + (1.06)^{-40}(0.29374)(0.23047) = 0.029874986
\]

\[
Var[L_n^0] = \left[ 40,000 + \frac{354.33}{0.06 / 1.06} \right] \left[ 0.029874986 - (0.13531704)^2 \right] = 24,747,243.94
\]
Part B:

\[ PVP = PVB \]

\[
12P\hat{\alpha}_{30}^{(12)} = 40,000\bar{A}_{30}^{(33)} \implies 12P[\hat{\alpha}_{30}^{(12)} - 40_{E_{30}} \cdot \bar{A}_{30}^{(12)}] = 40,000[\bar{A}_{30}^{(12)} - 40_{E_{30}} \cdot \bar{A}_{30}^{(12)} + 40_{E_{30}}]
\]

\[
12P[\alpha(12) \cdot \bar{a}_{30} - \beta(12) - 40_{E_{30}} \cdot (\alpha(12) \cdot \bar{a}_{30} - \beta(12))] = 40,000[(i \ell \delta)A_{40} - 40_{E_{30}} \cdot (i \ell \delta)A_{30} + 40_{E_{30}}]
\]

\[
P = \frac{40,000[(i \ell \delta)A_{40} - 40_{E_{30}} \cdot (i \ell \delta)A_{30} + 40_{E_{30}}]}{12[\alpha(12) \cdot \bar{a}_{30} - \beta(12) - 40_{E_{30}} \cdot (\alpha(12) \cdot \bar{a}_{30} - \beta(12))]} = \frac{40,000[(1.02971)(0.10248) - (0.29374)(0.23047)(1.02971)(0.51495) + (0.29374)(0.23047)]}{12[(1.00028)(15.8561) - 0.46812 - (0.29374)(0.23047)(1.00028)(8.5693) - 0.46812]} = 30.84
\]
7. Su, who is (70), buys a whole life insurance policy with a death benefit of 1,000,000 payable at the end of the year of death. Su will pay level annual premiums for this policy.

You are given:
   a. Mortality follows the Illustrative Life Table.
   b. \( i = 6\% \).
   c. Expenses are:
      i. Commissions of 55% of premium in the first year and 5% of premium thereafter.
      ii. Issue expenses of 1000 per policy plus 1.50 per 1000 of death benefit.
      iii. Maintenance expense at the beginning of every year of 50. This includes the first year.
      iv. Termination expense of 750 paid at the end of the year of death.

Calculate the annual gross premium that Su would pay under the equivalence principle.

Solution:

\[
PVP = PVB + PVE
\]

\[
P\alpha_{70} = 1,000,000A_{70} + 0.5P + 0.05P\alpha_{70} + 1000 + 1.50(1000) + 50\alpha_{70} + 750A_{70}
\]

\[
P = \frac{1,000,750A_{70} + 1000 + 1.50(1000) + 50\alpha_{70}}{\alpha_{70} - 0.05\alpha_{70} - 0.5} = \frac{1,000,750(0.51495) + 2500 + 50(8.5693)}{(0.95)(8.5693) - 0.5} = 67,828.28
\]
8. You are given:

   a. Mortality follows Gompertz law such that 
      \( \mu_{50} = e^{-0.0003 \ln(1.1) t} \) 
      \( \mu_{50} = e^{-0.0003 \ln(1.1) t} \)
   
   b. \( i = 6\% \)

The Wright Life Insurance Company uses this information and calculates that:

   a. \( A_{50} = 0.54039 \)
   
   b. \( \ddot{A}_{50} = 0.37837 \)

Wright collects a single net benefit premium of 500,000 from Erin, (50), for a whole life annuity due with level annual payments of \( C \). The single net benefit premium is the same as the actuarial present value of the annuity.

A. Calculate \( C \).

B. Calculate the variance of the present value random variable for this annuity.

C. Use the three term Woolhouse formula to determine the monthly benefit that Erin would have received if she had purchased a whole life annuity due with monthly payments.

Part A

\[ PVP = PVB \]

\[ P = C \ddot{a}_{50} \implies 500,000 = C \left( \frac{1 - A_{50}}{d} \right) = C \left( \frac{1 - 0.54039}{0.06 / 1.06} \right) = C(8.119776667) \]

\[ \therefore C = \frac{500,000}{8.119776667} = 61,578.05 \]

Part B

\[ \text{Var}[Y] = (61,578.05)^2 \left[ \frac{\ddot{A}_{50} - (A_{50})^2}{d^2} \right] = \]

\[ (61,578.05)^2 \left[ \frac{0.37837 - (0.54039)^2}{(0.06 / 1.06)^2} \right] = 102,191,932,400 \]
Part C

\[ PVP = PVB \]

\[ 500,000 = 12C \hat{\alpha}^{(2)}_{50} = 12C \left[ \hat{\alpha}_{50} - \frac{12 - 1}{2(12)} - \frac{12^2 - 1}{12(12^2)} (\mu_x + \delta) \right] \]

\[ \mu_x = Bc^x \implies \mu_{50} = (0.0003)(1.1)^{50} \]

\[ \therefore C = \frac{500,000}{12C \left[ 8.119776667 - \frac{11}{24} - \frac{143}{12^2} \right] ((0.0003)(1.1)^{50} + \ln(1.06))} = 5443.99 \]