Chapter 8

114. For a multiple state model where there are two states:

i. State 0 is a person is alive
ii. State 1 is a person is dead

Further you are given that a person can transition from State 0 to State 1 but not back again.

Using the Illustrative Life Table, determine the following:

a. $P_{00}^{10}$

b. $P_{01}^{10}$

c. $P_{10}^{10}$

115. For a multiple state model, there are three states:

i. State 0 is a person is healthy
ii. State 1 is a person is permanently disabled
iii. State 2 is a person is dead

Further you are given that a person can transition from State 0 to State 1 or State 2. Further, a person in State 1 can transition to State 2, but not to State 0. Finally, a person in State 2 cannot transition.

You are given the following transitional intensities:

i. $\mu_{1}^{01} = 0.05$

ii. $\mu_{1}^{02} = 0.01$

iii. $\mu_{2}^{12} = 0.02$

Calculate the following:

a. $P_{00}^{10}$

b. $P_{01}^{10}$

c. $P_{02}^{10}$
116. For a multiple state model, there are three states:

i. State 0 is a person is healthy
ii. State 1 is a person is sick
iii. State 2 is a person is dead

Further you are given that a person can transition from State 0 to State 1 or State 2. Further, a person in State 1 can transition to State 0 or to State 2. Finally, a person in State 2 cannot transition.

You are given the following transitional intensities:

i. $\mu_{01} = 0.05$
ii. $\mu_{10} = 0.03$
iii. $\mu_{02} = 0.01$
iv. $\mu_{12} = 0.02$

Calculate the following:

a. $P_{00}$

b. Assuming that only one transition can occur in any monthly period, use the Euler method to calculate:

i. $P_{00}$
ii. $P_{01}$
iii. $P_{00}$
iv. $P_{01}$
v. $P_{00}$
vi. $P_{01}$
117. For a multiple state model, there are three states:

   i. State 0 is a person is healthy
   ii. State 1 is a person is sick
   iii. State 2 is a person is dead

Further you are given that a person can transition from State 0 to State 1 or State 2. Also, a person in State 1 can transition to State 0 or to State 2. Finally, a person in State 2 cannot transition.

You are given the following transitional intensities:

   i. \( \mu_{x+1}^{01} = 0.05 + .001t \)
   ii. \( \mu_{x}^{10} = 0.03 - .0005t \)
   iii. \( \mu_{x}^{02} = 0.01 \)
   iv. \( \mu_{x}^{12} = 0.02 \)

Assume that only one transition can occur in any monthly period.

If \( 10 P_{x}^{00} = 0.90 \) and \( 10 P_{x}^{01} = 0.07 \), use the Euler method to calculate \( 121/12 P_{x}^{01} \).

118. *For a multiple state model, there are three states:

   i. State 0 is a person is healthy
   ii. State 1 is a person is sick
   iii. State 2 is a person is dead

Further you are given that a person can transition from State 0 to State 1 or State 2. Also, a person in State 1 can transition to State 0 or to State 2. Finally, a person in State 2 cannot transition.

You are given the following transitional intensities:

   v. \( \mu_{x+1}^{01} = 0.06 \)
   vi. \( \mu_{x}^{10} = 0.03 \)
   vii. \( \mu_{x}^{02} = 0.01 \)
   viii. \( \mu_{x}^{12} = 0.04 \)

Calculate the probability that a disabled life on July 1, 2012 will become healthy at some time before July 1, 2017 but will not remain continuously healthy until July 1, 2017.
119. * Employees in Purdue Life Insurance Company (PLIC) can be in:

i. State 0: Non-Executive employee
ii. State 1: Executive employee
iii. State 2: Terminated from employment

Emily joins PLIC as a non-executive employee at age 25.

You are given:

i. $\mu^{01} = 0.008$
ii. $\mu^{02} = 0.02$
iii. $\mu^{12} = 0.01$
iv. Executive employees never return to non-employee executive state.
v. Employees terminated from employment are never rehired.
vi. The probability that Emily lives for 30 years is 0.92, regardless of state.

Calculate the probability that Emily will be an executive employee of PLIC at age 55.
120. A person working for Organized Crime Incorporated (OCI) can have one of three statuses during their employment. The three statuses are:
   a. In Good Standing
   b. Out of Favor
   c. Dead

The following transitional probabilities indicates the probabilities of moving between the three status in an given year:

<table>
<thead>
<tr>
<th></th>
<th>In Good Standing</th>
<th>Out of Favor</th>
<th>Dead</th>
</tr>
</thead>
<tbody>
<tr>
<td>In Good Standing</td>
<td>0.6</td>
<td>0.3</td>
<td>0.1</td>
</tr>
<tr>
<td>Out of Favor</td>
<td>0.2</td>
<td>0.3</td>
<td>0.5</td>
</tr>
<tr>
<td>Dead</td>
<td>0</td>
<td>0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Calculate the probability that a person In Good Standing now will be Out of Favor at the end of the fourth year.

121. A person working for Organized Crime Incorporated (OCI) can have one of three statuses during their employment. The three statuses are:
   a. In Good Standing
   b. Out of Favor
   c. Dead

The following transitional probabilities indicates the probabilities of moving between the three status in an given year:

<table>
<thead>
<tr>
<th></th>
<th>In Good Standing</th>
<th>Out of Favor</th>
<th>Dead</th>
</tr>
</thead>
<tbody>
<tr>
<td>In Good Standing</td>
<td>0.6</td>
<td>0.3</td>
<td>0.1</td>
</tr>
<tr>
<td>Out of Favor</td>
<td>0.2</td>
<td>0.3</td>
<td>0.5</td>
</tr>
<tr>
<td>Dead</td>
<td>0</td>
<td>0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

At the beginning of the year, there are 1000 employees In Good Standing. All future states are assumed to be independent.

i. Calculate the expected number of deaths over the next four years.
ii. Calculate the variance of the number of the original 1000 employees who die within four years.
122. Animals species have three possible states: Healthy (row 1 and column 1 in the matrices), Endangered (row 2 and column 2 in the matrices), and Extinct (row 3 and column 3 in the matrices). Transitions between states vary by year where the subscript indicates the beginning of the year.

\[ Q_0 = \begin{pmatrix} 0.80 & 0.20 & 0 \\ 0 & 0.75 & 0.25 \\ 0 & 0 & 1 \end{pmatrix} \]

\[ Q_1 = \begin{pmatrix} 0.90 & 0.10 & 0 \\ 0.20 & 0.70 & 0.10 \\ 0 & 0 & 1 \end{pmatrix} \]

\[ Q_2 = \begin{pmatrix} 0.95 & 0.05 & 0 \\ 0.25 & 0.70 & 0.05 \\ 0 & 0 & 1 \end{pmatrix} \]

\[ Q_i = \begin{pmatrix} 0.95 & 0.05 & 0 \\ 0.3 & 0.70 & 0 \\ 0 & 0 & 1 \end{pmatrix} \]

for \( i > 2 \)

Calculate the probability that a species endangered at time 0 will become extinct.

123. A fully continuous whole life policy to (60) is subject to two decrements – Decrement 1 is death and Decrement 2 is lapse. The benefit upon death is 1000. No benefit is payable upon lapse.

You are given:

a. \( \mu_k^{(1)} = 0.015 \)

b. \( \mu_k^{(2)} = 0.100 \)

c. \( \delta = 0.045 \)

Calculate the \( P \), the premium rate payable annually.

124. A fully continuous whole life policy to (60) is subject to two decrements – Decrement 1 is death by accident and Decrement 2 is death by any other cause. The benefit upon death by accident is 2000. The death benefit upon death by any other cause is 1000.

You are given:

a. \( \mu_k^{(1)} = 0.015 \)

b. \( \mu_k^{(2)} = 0.025 \)

c. \( \delta = 0.06 \)

Calculate the \( P \), the premium rate payable annually.
125. For a multiple state model, there are three states:

i. State 0 is a person is healthy
ii. State 1 is a person is sick
iii. State 2 is a person is dead

Further you are given that a person can transition from State 0 to State 1 or State 2. Also, a person in State 1 can transition to State 0 or to State 2. Finally, a person in State 2 cannot transition.

You are given the following matrix of transitional probabilities:

\[
\begin{bmatrix}
0.80 & 0.15 & 0.05 \\
0.60 & 0.25 & 0.15 \\
0 & 0 & 1 \\
\end{bmatrix}
\]

A special 3-year term policy pays 500,000 at the end of the year of death. It also pays 100,000 at the end of the year if the insured is disabled.

Premiums are payable annually if the insured is healthy (state 0).

You are given $i = 10\%$.

i. Calculate the present value of the death benefits to be paid.
ii. Calculate the present value of the disability benefits to be paid.
iii. Calculate the annual benefit premium.
iv. Calculate the total reserve that would be held at the end of the first year.
v. Calculate the reserve associated with each person in state 0 at the end of the first year.
vi. Calculate the reserve associated with each person in state 1 at the end of the first year.
126. A person working for Organized Crime Incorporated (OCI) can have one of three statuses during their employment. The three statuses are:
   a. In Good Standing
   b. Out of Favor
   c. Dead

The following transitional probabilities indicates the probabilities of moving between the three status in an given year:

<table>
<thead>
<tr>
<th></th>
<th>In Good Standing</th>
<th>Out of Favor</th>
<th>Dead</th>
</tr>
</thead>
<tbody>
<tr>
<td>In Good Standing</td>
<td>0.6</td>
<td>0.3</td>
<td>0.1</td>
</tr>
<tr>
<td>Out of Favor</td>
<td>0.2</td>
<td>0.3</td>
<td>0.5</td>
</tr>
<tr>
<td>Dead</td>
<td>0</td>
<td>0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The Italian Life Insurance Company issues a special 4 year term insurance policy covering employees of OCI. The policy pays a death benefit of 10,000 at the end of the year of death.

Assume that the interest rate is 25% (remember who we are dealing with).

i. Calculate the actuarial present value of the death benefit for an employee who is In Good Standing at the issue of the policy.

ii. Calculate the annual benefit premium (paid at the beginning of the year by those in Good Standing and those Out of Favor) for an employee who is In Good Standing at the issue of the policy.

iii. Calculate the total reserve that Italian Life should hold at the end of the second year for a policy that was issued to an employee who was In Good Standing.

iv. Calculate the actuarial present value of the death benefit for an employee who is Out of Favor at the issue of the policy.

v. For an employee who is Out of Favor when the policy is issued, the annual contract premium payable at the beginning of each year that the employee is not Dead is 3500. Calculate the actuarial present value of the expected profit for Italian Life. The actuarial present value of the expected profit is the actuarial present value of the contract premiums less the actuarial present value of the death benefits.
A person working for Organized Crime Incorporated (OCI) can have one of three statuses during their employment. The three statuses are:

d. In Good Standing
e. Out of Favor
f. Dead

The following transitional probabilities indicates the probabilities of moving between the three status in an given year:

<table>
<thead>
<tr>
<th></th>
<th>In Good Standing</th>
<th>Out of Favor</th>
<th>Dead</th>
</tr>
</thead>
<tbody>
<tr>
<td>In Good Standing</td>
<td>0.6</td>
<td>0.3</td>
<td>0.1</td>
</tr>
<tr>
<td>Out of Favor</td>
<td>0.2</td>
<td>0.3</td>
<td>0.5</td>
</tr>
<tr>
<td>Dead</td>
<td>0</td>
<td>0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The Italian Life Insurance Company issues a special four year annuity covering employees of OCI. The annuity pays a benefit of 100,000 at the end of a year if the employee is In Good Standing at the end of the year. It pays a benefit of 50,000 if an employee is Out of Favor at the end of a year. No benefit is paid if the employee is Dead at the end of a year.

Assume that the interest rate is 25% (remember who we are dealing with).

i. Calculate the actuarial present value of the annuity benefit for an employee who is In Good Standing at the issue of the policy.

ii. Calculate the annual benefit premium (paid at the beginning of the year by those in Good Standing and those Out of Favor) for an employee who is In Good Standing at the issue of the policy.

iii. Calculate the reserve that Italian Life should hold at the end of the second year for a policy that was issued to an employee who was In Good Standing.
You are given the following table where decrement (1) is death, decrement (2) is lapse, and decrement (3) is diagnosis of critical illness:

<table>
<thead>
<tr>
<th>x</th>
<th>( q_x^{(1)} )</th>
<th>( q_x^{(2)} )</th>
<th>( q_x^{(3)} )</th>
<th>( p_x^{(r)} )</th>
<th>( l_x^{(r)} )</th>
<th>( d_x^{(1)} )</th>
<th>( d_x^{(2)} )</th>
<th>( d_x^{(3)} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>55</td>
<td>0.02</td>
<td>0.15</td>
<td>0.010</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>56</td>
<td>0.03</td>
<td>0.06</td>
<td>0.015</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>57</td>
<td>0.04</td>
<td>0.04</td>
<td>0.020</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>58</td>
<td>0.05</td>
<td>0.03</td>
<td>0.025</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>59</td>
<td>0.06</td>
<td>0.02</td>
<td>0.030</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

a. Complete the table using a radix of 10,000.
b. Calculate:
   i. \( 3 p_{55}^{(r)} \)
   ii. \( 2 q_{56}^{(2)} \)
   iii. \( q_{55}^{(3)} \)
   iv. The probability that a person age 55 will decrement from death or critical illness before age 60.

c. Assuming uniform distribution of each decrement between integer ages, calculate:
   i. \( 0.25 q_{55}^{(2)} \)
   ii. \( 0.5 p_{56}^{(r)} \)
   iii. \( 0.5 p_{56.8}^{(1)} \)
   iv. \( 0.5 q_{55.6}^{(1)} \)

d. Assuming a constant force of decrement for each decrement between integer ages, calculate:
   i. \( 0.25 q_{55}^{(2)} \)
   ii. \( 0.5 p_{56}^{(r)} \)
   iii. \( 0.5 p_{56.8}^{(1)} \)
   iv. \( 0.5 q_{55.6}^{(1)} \)
129. A fully discrete 3 year term pays a benefit of 1000 upon any death. It pays an additional 1000 (for a total of 2000) upon death from accident. You are given:

<table>
<thead>
<tr>
<th>x</th>
<th>$q_x^{(1)}$</th>
<th>$q_x^{(2)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.030</td>
<td>0.010</td>
</tr>
<tr>
<td>21</td>
<td>0.025</td>
<td>0.020</td>
</tr>
<tr>
<td>22</td>
<td>0.020</td>
<td>0.030</td>
</tr>
</tbody>
</table>

Decrement (1) is death from accidental causes while decrement (2) is death from non-accidental causes.

The annual effective interest rate is 10%.

a. Calculate the level annual net premium for this insurance.
b. Calculate the net premium reserve at the end of year 0, 1, 2, and 3.

130. You are given:

a. $q_x^{(1)} = 0.200$
b. $q_x^{(2)} = 0.080$
c. $q_x^{(3)} = 0.125$

Assuming that each decrement is uniformly distributed over each year of age in the associated single decrement table, calculate $q_x^{(1)}$.

131. You are given:

a. $q_x^{(1)} = 0.200$
b. $q_x^{(2)} = 0.080$
c. $q_x^{(3)} = 0.125$

Assuming that each decrement in the multiple decrement table is uniformly distributed over each year of age, calculate $q_x^{(1)}$.

132. You are given the following for a double decrement table:

a. $q_x^{(1)} = 0.200$
b. $q_x^{(2)} = 0.080$

Assuming that each decrement in the multiple decrement table is uniformly distributed over each year of age, calculate $0.4q_x^{(1)+0.4}$. 

November 19, 2012
133. You are given:
   a. \( q_{x}^{(1)} = 0.200 \)
   b. \( q_{x}^{(2)} = 0.080 \)

   Decrement (1) is uniformly distributed over the year. Decrement (2) occurs at time 0.6.

   Calculate \( q_{x}^{(2)} \).

134. For a double decrement table with \( l_{40}^{1} = 2000 \):

<table>
<thead>
<tr>
<th>x</th>
<th>( q_{x}^{(1)} )</th>
<th>( q_{x}^{(2)} )</th>
<th>( q_{x}^{1} )</th>
<th>( q_{x}^{2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>40</td>
<td>0.24</td>
<td>0.10</td>
<td>0.25</td>
<td>y</td>
</tr>
<tr>
<td>41</td>
<td>--</td>
<td>--</td>
<td>0.20</td>
<td>2y</td>
</tr>
</tbody>
</table>

   Calculate \( l_{42}^{1} \).

135. For iPhones, the phone may cease service for mechanical failure or for other reasons (lost, stolen, dropped in a pitcher of beer, etc). You are given the following double decrement table:

<table>
<thead>
<tr>
<th>Year of Service</th>
<th>For an iPhone at the beginning of the year of service, probability of</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mechanical Failure</td>
</tr>
<tr>
<td>1</td>
<td>0.2</td>
</tr>
<tr>
<td>2</td>
<td>--</td>
</tr>
<tr>
<td>3</td>
<td>--</td>
</tr>
</tbody>
</table>

   You are also given:
   a. The number of iPhones that terminate for other reasons in year 3 is 40% of the number of iPhones that survive to the end of year 2.
   b. The number of iPhones that terminate for other reasons in year 2 is 80% of the number of iPhones that survive to the end of year 2.

   Calculate the probability that an iPhone will cease to function due to mechanical failure during the three year period following its entry into service.
136. *Your actuarial student has constructed a multiple decrement table using independent mortality and lapse tables. The multiple decrement table values, where decrement $d$ is death and decrement $w$ is lapse, are as follows:

<table>
<thead>
<tr>
<th>$l_{60}$</th>
<th>$d_{60}^{(d)}$</th>
<th>$d_{60}^{(w)}$</th>
<th>$l_{61}^{(r)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>950,000</td>
<td>2,580</td>
<td>94,742</td>
<td>852,678</td>
</tr>
</tbody>
</table>

You discover that an incorrect value of $q_{60}^{(w)}$ was taken from the independent lapse table. The correct value is 0.05.

Decrements are uniformly distributed over each year of age in the multiple decrement table.

You correct the multiple decrement table, keeping $l_{60}^{(r)} = 950,000$.

Calculate the correct values of $d_{60}^{(w)}$. 

November 19, 2012
137. You are given that mortality follows the Illustrative Life Table with $i = 0.06$. Assuming that lives are independent and that deaths are uniformly distributed between integral ages, calculate:

a. $10q_{50:60}$

b. $10q_{50:60}$

c. $A_{60:60}$

d. The net annual premium for a fully discrete joint whole life on (60) and (60) with a death benefit of 1000.

e. $\ddot{A}_{60:60}$

f. The net annual premium rate for a fully continuous joint whole life on (60) and (60) with a death benefit of 1000.

g. $10p_{50:60}$

h. Calculate the probability that the survivor of (50) and (60) dies in year 11.

i. Calculate the probability that exactly one life of (50) and (60) is alive after 10 years.

j. $A_{60:70}$

k. The net annual premium rate for a fully discrete survivor whole life on (60) and (70) with a death benefit of 1000.

l. $\ddot{a}_{40:60}$

m. $\ddot{a}_{40:70}$

n. $\ddot{a}_{40:60}$

138. An joint annuity on (50) and (60) pays a benefit of 1 at the beginning of each year if both annuitants are alive. The annuity pays a benefit of 2/3 at the beginning of each year if one annuitant is alive.

You are given:

i. Mortality follows the Illustrative Life Table.

ii. (50) and (60) are independent lives.

iii. $i = 0.06$

Calculate the actuarial present value of this annuity.
139. An joint annuity on (50) and (60) pays a benefit of 1 at the beginning of each year if both annuitants are alive. The annuity pays a benefit of \(2/3\) at the beginning of each year if only (50) is alive. The annuity pays a benefit of \(1/2\) at the beginning of each year if only (60) is alive.

You are given:

i. Mortality follows the Illustrative Life Table.
ii. (50) and (60) are independent lives.
iii. \(i = 0.06\)

Calculate the actuarial present value of this annuity.

140. You are given the following mortality table:

<table>
<thead>
<tr>
<th>(x)</th>
<th>(l_x)</th>
<th>(q_x)</th>
<th>(p_x)</th>
</tr>
</thead>
<tbody>
<tr>
<td>90</td>
<td>1000</td>
<td>0.10</td>
<td>0.90</td>
</tr>
<tr>
<td>91</td>
<td>900</td>
<td>0.20</td>
<td>0.80</td>
</tr>
<tr>
<td>92</td>
<td>720</td>
<td>0.40</td>
<td>0.60</td>
</tr>
<tr>
<td>93</td>
<td>432</td>
<td>0.50</td>
<td>0.50</td>
</tr>
<tr>
<td>94</td>
<td>216</td>
<td>1.00</td>
<td>0.00</td>
</tr>
<tr>
<td>95</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Assume that deaths are uniformly distributed between integral ages and the lives are independent. Calculate at \(i = 4\%\):

a. \(A_{91:92}\)

b. \(A_{90:91:93}\)

c. \(\bar{a}_{92:93}\)
141. * You are given:

i. \( 3p_{40} = 0.990 \)
ii. \( 6p_{40} = 0.980 \)
iii. \( 9p_{40} = 0.965 \)
iv. \( 12p_{40} = 0.945 \)
v. \( 15p_{40} = 0.920 \)
vi. \( 18p_{40} = 0.890 \)

For two independent lives aged 40, calculate the probability that the first death occurs after 6 years, but before 12 years.

142. * For a last survivor insurance of 10,000 on independent lives (70) and (80), you are given:

i. The benefit, payable at the end of the year of death, is paid only if the second death occurs during year 5.
ii. Mortality follows the Illustrative Life Table
iii. \( i = 0.03 \)

Calculate the actuarial present value of this insurance.

143. * For a temporary life annuity-immediate on independent lives (30) and (40):

i. Mortality follows the Illustrative Life Table
ii. \( i = 0.06 \)

Calculate \( a_{30:40\overline{70}} \)
144. Two lives, (x) and (y), are subject to a common shock model. You are given:

i. \( \mu_x(t) = 0.04 \)

ii. \( \mu_y(t) = 0.06 \)

iii. \( \mu_x(t) \) and \( \mu_y(t) \) do not reflect the mortality from the common shock.

iv. The mortality from the common shock is a constant force of 0.01.

v. \( \delta = 0.03 \)

Calculate:

a. \( p_x^o \)

b. \( p_y^o \)

c. \( p_{xy}^o \)

d. \( e_x^o \)

e. \( e_y^o \)

f. \( e_{xy}^o \)

g. \( e_{xy}^o \)

h. \( \overline{A}_{xy} \)

i. \( \overline{A}_{xy} \)
145. You are given:

i. \( \mu_x(t) = 0.02t \)

ii. \( \mu_x(t) = (40 - t)^{-1} \)

iii. The lives are subject to an common shock model with \( \mu = 0.015 \)

iv. \( \mu_x(t) \) and \( \mu_y(t) \) incorporate deaths from the common shock.

Calculate \( 3q_{xy} \)

146. A male age 65 and a female age 60 are subject a common shock model.

You are given:

i. Male mortality follows the Illustrative Life Table.

ii. Female mortality follows the Illustrative Life Table but for a live five years younger. In other words, the mortality for a female age 60 is equal to the mortality for a person 55 in the Illustrative Life Table.

iii. The above mortality rates do not reflect the common shock risk.

iv. Both lives are subject to a common shock based on a constant force of mortality of 0.01.

v. \( i = 8\% \) - NOTE that it is not 6%.

Calculate:

a. \( 20E_{65} \) where (65) is a male.

b. \( 20E_{60} \) where (60) is a female.

c. \( 20E_{65:60} \) where (65) is a male and (60) is a female.

d. \( 20E_{65:60} \) where (65) is a male and (60) is a female.

147. * You are pricing a special 3-year annuity-due on two independent lives, both age 80. The annuity pays 30,000 if both persons are alive and 20,000 if only one person is alive.

You are given that \( i = 0.05 \), \( 1p_{80} = 0.91 \), \( 2p_{80} = 0.82 \) and \( 3p_{80} = 0.72 \).

Calculate the actuarial present value of this annuity.
148. *For a special continuous joint life annuity on \(( x \) and \(( y \), you are given:

i. The annuity payments are 25,000 per year while both are alive and 15,000 per year when only one is alive.

ii. The annuity also pays a death benefit of 30,000 upon the first death.

iii. \( i = 0.06 \)

iv. \( a_{xy} = 8 \)

v. \( a_{xy} = 10 \)

Calculate the actuarial present value of this special annuity.
Answers

114.
   a. 0.27041
   b. 0.72959
   c. 0.00000

115.
   a. 0.54881
   b. 0.33740
   c. 0.11379

116.
   a. 0.54881
   b.
      i. 1.00000
      ii. 0.00000
      iii. 0.99500
      iv. 0.004167
      v. 0.990035
      vi. 0.008295

117. 0.074238
118. 0.0209
119. 0.12639
120. 0.1539
121.
   i. 621.1
   ii. 235.33479
122. 0.35125
123. 15
124. 55
125.
   i. 71.140.12
   ii. 37,838.09
   iii. 46,760.15
   iv. 12,038.07
   v. -5,560.92
   vi. 105,899.42
126.
   i. 3599.51
   ii. 1484.79
   iii. 928.08
   iv. 5823.44
   v. 240.46
127.
   i. 128,854.27
   ii. 53,152.09
   iii. -9,280.82

128.
   a.

   \[
   \begin{array}{cccccccc}
   x & q_s^{(1)} & q_s^{(2)} & q_s^{(3)} & p_x^{(z)} & l_x^{(z)} & d_s^{(1)} & d_s^{(2)} & d_s^{(3)} \\
   55 & 0.02 & 0.15 & 0.010 & 0.180 & 0.820 & 10,000 & 200 & 1500 & 100 \\
   56 & 0.03 & 0.06 & 0.015 & 0.105 & 0.895 & 8,200 & 246 & 492 & 123 \\
   57 & 0.04 & 0.04 & 0.020 & 0.100 & 0.900 & 7,339 & 293.56 & 293.56 & 146.78 \\
   58 & 0.05 & 0.03 & 0.025 & 0.105 & 0.895 & 6,605.1 & 330.255 & 198.153 & 165.1275 \\
   59 & 0.06 & 0.02 & 0.030 & 0.110 & 0.890 & 5,911.5645 & 354.69387 & 118.23129 & 177.346935 \\
   \end{array}
   \]

   b.
   i. 0.66051
   ii. 0.0958
   iii. 0.026978
   iv. 0.21368

   c.
   i. 0.0375
   ii. 0.9475
   iii. 0.94776
   iv. 0.01173

   d.
   i. 0.04034
   ii. 0.94604
   iii. 0.94763
   iv. 0.01139

129.
   a. 63.64
   b. The reserve is zero at the end of each year.

130. 0.180167
131. 0.180520
132. 0.085951
133. 0.0704
134. 802.56
135. 0.35
136. 47,433
137.
   a. 0.26084
b. 0.03436
c. 0.47975
d. 52.20
e. 0.49400
f. 56.89
g. 0.98364
h. 0.00504
i. 0.24448
j. 0.31180
k. 25.65
l. 13.0997
m. 12.1584
n. 3.5891
138. 12.8768
139. 12.7182
140.
  a. 0.93867
  b. 0.83045
  c. 1.28846
141. 0.067375
142. 234.82
143. 7.1687
144.
  a. $e^{-0.05t}$
  b. $e^{-0.07t}$
  c. $e^{-0.11t}$
  d. 20
  e. 14.286
  f. 9.091
  g. 25.195
  h. 0.78571
  i. 0.53929
145. 0.06994
146.
  a. 0.05498
  b. 0.10970
  c. 0.03434
  d. 0.13034
147. 80,431.70
148. 246,015.46