There are two Versims of This Test

FIND Your Versim.

STAT 472
Fall 2018
Test 1

October 2, 2018

1. (10 points) You are given that $F_{50}(t) = 0.0004t^2$ for $0 \le t \le 50$ and $\delta = 0.05$.

Calculate $1000\bar{A}_{50}$.

Solutions:

$S_{50}(t) = {}_t p_x = 1 - 0.0004t^2$

$$1000\bar{A}_{50} = 1000\int_0^{50} v^t \cdot {}_t p_{50} \cdot \mu_{50+t} \cdot dt$$

$$\mu_{50+t} = \frac{-\frac{d}{dt}\,{}_t p_x}{{}_t p_x} = \frac{0.0008t}{{}_t p_x}$$

$$1000\int_0^{50} v^t \cdot {}_t p_{50} \cdot \mu_{50+t} \cdot dt = 1000\int_0^{50} v^t (0.0008t)dt = 0.8\int_0^{50} te^{-0.05t}dt$$

$$u = t \implies du = dt \quad \text{and} \quad dv = e^{-0.05t} \implies v = -\frac{e^{-0.05t}}{0.05}$$

$$0.8\int_0^{50} te^{-0.05t}dt = 0.8\left(\left[-\frac{e^{-0.05t}}{0.05}\right]_0^{50} + \int_0^{50}\frac{e^{-0.05t}}{0.05}dt\right) = 0.8\left(-82.0850 + \left[-\frac{e^{-0.05t}}{(0.05)^2}\right]_0^{50}\right)$$

$$0.8(-82.0850 - 32.8340 + 400) = 228.0648$$

2. (10 points) You are given the following mortality table:

x	q_x
90	0.1
91	0.3
92	0.5
93	0.7
94	1.0

For ages 90 to 91 and 91 to 92, deaths are uniformly distributed between integral ages.

For ages 92 to 93 and 93 to 94, there is a constant force of mortality between integral ages.

Calculate ${}_{0.8|1.5}q_{90.6}$.

Solution:

$${}_{0.8|1.5}q_{90.6} = \frac{l_{91.4} - l_{92.9}}{l_{90.6}}$$

$$l_{90} = 1000; l_{91} = (1000)(1-0.1) = 900; l_{92} = (900)(1-0.3) = 630; l_{93} = (630)(1-0.5) = 315$$

$$l_{90.6} = (1-0.6)(1000) + (0.6)(900) = 940$$
$$l_{91.4} = (1-0.4)(900) + (0.4)(630) = 792$$
$$l_{92.9} = (630)^{(1-0.9)}(315)^{0.9} = 337.60864$$

$${}_{0.8|1.5}q_{90.6} = \frac{l_{91.4} - l_{92.9}}{l_{90.6}} = \frac{792 - 337.60864}{940} = 0.4834$$

3. (10 points) You are given the following two year select and ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
100	0.20	0.40	0.70	102
101	0.30	0.60	1.00	103

Calculate $Var[K_{[100]}]$

Solution:

$$Var[K_{[100]}] = 2\sum k \cdot_k p_x - e_x - (e_x)^2$$

$$e_{[100]} = p_{[100]} +_2 p_{[100]} +_3 p_{[100]} = (0.8) + (0.8)(0.6) + (0.8)(0.6)(0.3) = 1.424$$

$$\sum k \cdot_k p_x = (1)(0.8) + (2)(0.8)(0.6) + (3)(0.8)(0.6)(0.3) = 2.192$$

$$Var = (2)(2.192) - 1.424 - (1.424)^2 = 0.9322$$

4. (6 points) Compare and contrast aggregate survival models with select and ultimate mortality models. In other words, state how they are the same and how they are different. Also, state when each should be used.

   Solution:

   Both aggregate models and select and ultimate models represent a mortality model of their underlying population. The aggregate model is based on populations where underwriting has not taken place. The only variable in determining the expected mortality is the age of the life. On the other hand, when underwriting has taken place such as with an insured population, then the mortality will be a function of both the age at underwriting and the number of year that have passed since underwriting has occurred. It is not appropriate to use aggregate tables where underwriting has occurred.

5. Knapp Industries uses robots to produce their products. Improving maintenance protocols will extend the lifetime of an industrial robot. The robot's mortality rates and improvement factors are given below:

x	$q(x,0)$	$\varphi(x,1)$	$\varphi(x,2)$
0	0.1	0.30	0.22
1	0.2	0.25	0.15
2	0.4	0.10	0.08

Knapp buys 1000 robots.

a. (10 points) Complete the following table being sure to show your work:

x	l_x
0	1000
1	$l_1 = 1000(1-q(0,0)) = (1000)(0.9) = 900$
2	$l_2 = 900(1-q(1,1)) = (900)(1-(0.2)(1-0.25)) = 765$
3	$l_2 = 765(1-q(2,2)) = (765)(1-(0.4)(1-0.1)(1-0.08)) = 511.632$

Knapp decides to buy a three year warranty on each new robot that will pay a benefit of 100,000 at the end of the year if a robot dies. (This is equivalent to a 3 year term policy on the robot's life.)

b. (10 points) Calculate the Expected Present Value of the three year warranty using $d = 0.10$. This value should be for each robot, not for all 1000 robots.

Solution:

$$v = 1 - d = 1 - 0.1 = 0.9$$

$$1000A^{1}_{0:\overline{3}|} = v(1000-900) + v^2(900-765) + v^3(765-511.632)$$

$$A^{1}_{0:\overline{3}|} = 0.3840553$$

$$100{,}000A^{1}_{0:\overline{3}|} = 38{,}405.53$$

6. You are given that mortality follows the Standard Ultimate Life Table with interest at 5%. You are also given that deaths are uniformly distributed between integral ages.

   a. (10 points) Calculate the Expected Present Value for a 30 year term insurance issued to (50) with a death benefit of 10,000 paid **at the end of the year of death**.

      Solution:

      $$10,000A^{\ 1}_{50:\overline{30|}} = 10,000(A_{50} - {}_{30}E_{50} \cdot A_{80})$$

      $$= (10,000)(0.18931 - (0.34824)(0.50994)(0.59293)) = 840.1645$$

   b. (10 points) Calculate the $Var[Z]$ where Z is the present value random variable for a 30 year term insurance issued to (50) with a death benefit of 10,000 paid **at the end of the year of death.**

      Solution:

      $$Var = (10,000)^2\left[{}^2A^{\ 1}_{50:\overline{30|}} - \{A^{\ 1}_{50:\overline{30|}}\}^2\right]$$

      $${}^2A^{\ 1}_{50:\overline{30|}} = {}^2A_{50} - {}^2{}_{30}E_{50} \cdot {}^2A_{80} = 0.05108 - (1.05)^{-30}(0.34824)(0.50994)(0.38134)$$

      $$= 0.03541$$

      $$Var = (10,000)^2\left[{}^2A^{\ 1}_{50:\overline{30|}} - \{A^{\ 1}_{50:\overline{30|}}\}^2\right] = (10,000)^2\left[0.03541 - (0.08401645)^2\right]$$

      $$= 2,835,258.1$$

c. (10 points) Calculate the Expected Present Value for a 30 year endowment insurance issued to (50) with a death benefit of 10,000 **paid at the moment of death**.

Solution:

$$10,000\bar{A}_{50:\overline{30|}} = (10,000)\left[\left(\frac{i}{\delta}\right)A^{\,1}_{50:\overline{30|}} + {}_{30}E_{50}\right]$$

$$= (10,000)[(1.02480)(0.08401645) + (0.50994)(0.59293)] = 2636.8181$$

d. (4 points) Explain why the Expected Present Value in c. is greater than the Expected Present Value in a. Provide two reasons.

Solution:

Endowment insurance will pay during the first 30 years if someone dies and will also pay at the end of 30 years to anyone that is alive. Term insurance only pays to those that die during the first 30 years.

Part c pays at the moment of death while part a pays at the end of the year of death. This means that part c pays earlier that part a so the present value is larger.

7. (10 points) Let Z_x be the present value random variable for a whole life insurance policy on (x) with a death benefit of 1 payable at the end of the year of death.

You are given:

a. $A_{50} = 0.300$

b. $Var[Z_{50}] = 0.110$

c. $q_{50} = 0.01$

d. $v = 0.93$

Calculate the $Var[Z_{51}]$.

Solution:

$$Var[Z_{50}] = {}^2A_{50} - (A_{50})^2 ==> 0.11 = {}^2A_{50} - (0.3)^2 ==> {}^2A_{50} = 0.2$$

$$A_{50} = vq_{50} + vp_{50}A_{51}$$

$$0.3 = (0.93)(0.01) + (0.93)(0.99)(A_{51}) ==> A_{51} = \frac{0.3 - 0.0093}{(0.93)(0.99)} = 0.31574$$

$${}^2A_{50} = v^2q_{50} + v^2p_{50} \cdot {}^2A_{51}$$

$$0.2 = (0.93)^2(0.01) + (0.93)^2(0.99)({}^2A_{51}) ==> {}^2A_{51} = \frac{0.2 - (0.93)^2(0.01)}{(0.93)^2(0.99)} = 0.22348$$

$$Var[Z_{51}] = 0.22348 - (0.31574)^2 = 0.1238$$

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1. (10 points) Let Z_x be the present value random variable for a whole life insurance policy on (x) with a death benefit of 1 payable at the end of the year of death.

   You are given:

   a. $A_{50} = 0.400$

   b. $Var[Z_{50}] = 0.140$

   c. $q_{50} = 0.015$

   d. $v = 0.93$

   Calculate the $Var[Z_{51}]$.

   Solution:

   $$Var[Z_{50}] = {}^2A_{50} - (A_{50})^2 \implies 0.14 = {}^2A_{50} - (0.4)^2 \implies {}^2A_{50} = 0.3$$

   $$A_{50} = vq_{50} + vp_{50}A_{51}$$

   $$0.4 = (0.93)(0.015) + (0.93)(0.985)(A_{51}) \implies A_{51} = \frac{0.3 - 0.93(0.015)}{(0.93)(0.985)} = 0.4214$$

   $${}^2A_{50} = v^2q_{50} + v^2p_{50} \cdot {}^2A_{51}$$

   $$0.3 = (0.93)^2(0.015) + (0.93)^2(0.985)({}^2A_{51}) \implies {}^2A_{51} = \frac{0.2 - (0.93)^2(0.015)}{(0.93)^2(0.9859)} = 0.3369$$

   $$Var[Z_{51}] = 0.3369 - (0.4214)^2 = 0.15931$$

2. (10 points) You are given that $F_{50}(t)=0.0004t^2$ for $0\le t\le 50$ and $\delta=0.06$.

Calculate $1000\bar{A}_{50}$.

Solutions:

$$S_{50}(t) = {}_t p_x = 1-0.0004t^2$$

$$1000\bar{A}_{50} = 1000\int_0^{50} v^t \cdot {}_t p_{50}\cdot \mu_{50+t}\cdot dt$$

$$\mu_{50+t} = \frac{-\frac{d}{dt}\,{}_t p_x}{{}_t p_x} = \frac{0.0008t}{{}_t p_x}$$

$$1000\int_0^{50} v^t \cdot {}_t p_{50}\cdot \mu_{50+t}\cdot dt = 1000\int_0^{50} v^t(0.0008t)dt = 0.8\int_0^{50} te^{-0.06t}dt$$

$$u=t \text{ ==> } du=dt \quad \text{and} \quad dv=e^{-0.06t} \text{ ==> } v=-\frac{e^{-0.06t}}{0.06}$$

$$0.8\int_0^{50} te^{-0.05t}dt = 0.8\left(\left[-\frac{e^{-0.06t}}{0.06}\right]_0^{50}+\int_0^{50}\frac{e^{-0.06t}}{0.06}dt\right) = 0.8\left(-41.48922+\left[-\frac{e^{-0.06t}}{(0.06)^2}\right]_0^{50}\right)$$

$$0.8(-41.48922-13.82974+277.778)=177.965$$

3. You are given that mortality follows the Standard Ultimate Life Table with interest at 5%. You are also given that deaths are uniformly distributed between integral ages.

   a. (10 points) Calculate the Expected Present Value for a 30 year term insurance issued to (65) with a death benefit of 10,000 paid **at the end of the year of death**.

      Solution:

      $$10,000A^{\ 1}_{65:\overline{30|}} = 10,000(A_{65} - {}_{30}E_{65} \cdot A_{95})$$

      $$= (10,000)(0.35477 - (0.24381)(0.21250)(0.81897)) = 3123.39$$

   b. (10 points) Calculate the $Var[Z]$ where Z is the present value random variable for a 30 year term insurance issued to (65) with a death benefit of 10,000 paid **at the end of the year of death.**

      Solution:

      $$Var = (10,000)^2\left[{}^2A^{\ 1}_{65:\overline{30|}} - \{A^{\ 1}_{65:\overline{30|}}\}^2\right]$$

      $${}^2A^{\ 1}_{65:\overline{30|}} = {}^2A_{65} - {}^2_{30}E_{65} \cdot {}^2A_{65} = 0.15420 - (1.05)^{-30}(0.24381)(0.21250)(0.68209)$$

      $$= 0.146023$$

      $$Var = (10,000)^2\left[{}^2A^{\ 1}_{50:\overline{30|}} - \{A^{\ 1}_{50:\overline{30|}}\}^2\right] = (10,000)^2\left[0.15420 - (0.312339)^2\right]$$

      $$= 4,846,768$$

c. (10 points) Calculate the Expected Present Value for a 30 year endowment insurance issued to (65) with a death benefit of 10,000 **paid at the moment of death.**

Solution:

$$10{,}000\bar{A}_{65:\overline{30|}} = (10{,}000)\left[\left(\frac{i}{\delta}\right)A^{\;1}_{65:\overline{30|}} + {}_{30}E_{65}\right]$$

$$= (10{,}000)[(1.02480)(0.31233947) + (0.24381)(0.21250] = 3718.95$$

d. (4 points) Explain why the Expected Present Value in c. is greater than the Expected Present Value in a. Provide two reasons.

Solution:

Endowment insurance will pay during the first 30 years if someone dies and will also pay at the end of 30 years to anyone that is alive. Term insurance only pays to those that die during the first 30 years.

Part c pays at the moment of death while part a pays at the end of the year of death. This means that part c pays earlier that part a so the present value is larger.

4. (10 points) You are given the following mortality table:

x	q_x
90	0.1
91	0.3
92	0.5
93	0.8
94	1.0

For ages 90 to 91 and 91 to 92, there is a constant force of mortality between integral ages.

For ages 92 to 93 and 93 to 94, deaths are uniformly distributed between integral ages.

Calculate ${}_{1.5|0.5}q_{90.8}$.

Solution:

$${}_{1.5|0.5}q_{90.8} = \frac{l_{92.3} - l_{92.8}}{l_{90.8}}$$

$$l_{90} = 1000; l_{91} = (1000)(1-0.1) = 900; l_{92} = (900)(1-0.3) = 630; l_{93} = (630)(1-0.5) = 315$$

$$l_{90.8} = (1000)^{(1-0.8)}(900)^{0.8} = 919.1661188$$
$$l_{92.3} = (1-0.3)(630) + (0.3)(315) = 535.5$$
$$l_{92.8} = (1-0.8)(630) + (0.8)(315) = 378$$

$${}_{0.8|1.5}q_{90.6} = \frac{l_{91.4} - l_{92.9}}{l_{90.6}} = \frac{535.5 - 378}{919.1661188} = 0.17135$$

5. (10 points) You are given the following two year select and ultimate mortality table:

$[x]$	$q_{[x]}$	$q_{[x]+1}$	q_{x+2}	$x+2$
100	0.20	0.40	0.60	102
101	0.30	0.50	1.00	103

Calculate $Var[K_{[100]}]$

Solution:

$$Var[K_{[100]}] = 2\sum k \cdot_k p_x - e_x - (e_x)^2$$

$$e_{[100]} = p_{[100]} +_2 p_{[100]} +_3 p_{[100]} = (0.8) + (0.8)(0.6) + (0.8)(0.6)(0.4) = 1.472$$

$$\sum k \cdot_k p_x = (1)(0.8) + (2)(0.8)(0.6) + (3)(0.8)(0.6)(0.4) = 2.336$$

$$Var = (2)(2.336) - 1.472 - (1.472)^2 = 1.033216$$

6. (3 points) Explain why population tables are not appropriate to use with life insurance products.

   Solution:

   Population mortality tables reflect the mortality of a population as a whole. Those that buy insurance have better mortality than the population as a whole for a number of reasons. The primary reason is that lives that buy insurance are underwritten. Additionally, even without the underwriting, insured lives have better mortality due to wealth, better healthcare, etc. If you use population mortality, you will overestimate the mortality of the insured lives.

7. (3 points) Explain why the mortality during the select period is less than the mortality during ultimate period in a select and ultimate mortality table.

   Solution:

   Mortality during the select period is less because insured lives are underwritten. Underwriting assures that a person is in good health at the time that a policy issued. Overtime, the effect of underwriting wears off. Therefore, after the select period, it is assumed that the underwriting no longer effects the morality and the insured enters the ultimate period.

8. Knapp Industries uses robots to produce their products. Improving maintenance protocols will extend the lifetime of an industrial robot. The robot's mortality rates and improvement factors are given below:

x	$q(x,0)$	$\varphi(x,1)$	$\varphi(x,2)$
0	0.1	0.25	0.22
1	0.2	0.20	0.17
2	0.4	0.15	0.10

Knapp buys 1000 robots.

a. (10 points) Complete the following table being sure to show your work:

x	l_x
0	1000
1	$l_1 = 1000(1-q(0,0)) = (1000)(0.9) = 900$
2	$l_2 = 900(1-q(1,1)) = (900)(1-(0.2)(1-0.2)) = 756$
3	$l_2 = 756(1-q(2,2)) = (756)(1-(0.4)(1-0.15)(1-0.10)) = 524.664$

Knapp decides to buy a three year warranty on each new robot that will pay a benefit of 100,000 at the end of the year if a robot dies. (This is equivalent to a 3 year term policy on the robot's life.)

b. (10 points) Calculate the Expected Present Value of the three year warranty using $d = 0.10$. This value should be for each robot, not for all 1000 robots.

Solution:

$$v = 1 - d = 1 - 0.1 = 0.9$$

$$1000 A^{1}_{0:\overline{3}|} = v(1000-900) + v^2(900-756) + v^3(765-524.664)$$

$$A^{1}_{0:\overline{3}|} = 0.3752839$$

$$100{,}000 A^{1}_{0:\overline{3}|} = 37{,}528.39$$