STAT 475
Quiz 2
Spring 2018

March 6, 2018

1. The Bloom Life Insurance Company sells a whole life insurance policy to (60). The policy pays a death benefit of 100,000 at the end of the year of death. The policy has annual premiums paid at the beginning of each policy year.

   You are given:

   i. Mortality follows the Illustrative Life Table.

   ii. Interest rates are uncertain but are distributed as follows:

      1. 5% with a probability of 30%
      2. 6% with a probability of 40%
      3. 7% with a probability of 30%

   iii. Net annual premiums are determined using 6% which is the expected value of the interest rate.

   iv. The following table of values:

i	d	A_{60}	${}^2A_{60}$
5%	0.04762	0.42580	0.21952
6%	0.05660	0.36913	0.17741
7%	0.06542	0.32268	0.14669

   a. (2 points) The net annual premium is 3300 to the nearest 100. Calculate it to the nearest 1.

   Solution:

$$PVP = PVB$$
$$P\ddot{a}_{60} = 100{,}000A_{60}$$

$$\ddot{a}_{60} = \frac{1-A_{60}}{d} = \frac{1-0.36913}{0.0566} = 11.14611$$

$$P = \frac{(100{,}000)(0.36913)}{11.14611} = 3312$$

The loss at issue random variable for this policy is L_0.

b. (12 points) Complete the following table. Show your work.

i	$E[L_0 \mid i]$	$Var[L_0 \mid i]$
5%	$(100,000)(0.4258)$ $-3312\left(\frac{1-0.4258}{0.04762}\right)$ $= 2644.04$	$\left(S+\frac{P}{d}\right)^2 ({}^2A_{60} - [A_{60}]^2)$ $\left(100,000+\frac{3312}{0.04762}\right)^2 (0.21952 - [0.4258]^2)$ $(169,550.6)^2 (0.03821)$ $= 1,098,563,720$
6%	0 Due to Equivalence Principle	$(158,515.9)^2 (0.041153)$ $= 1,034,064,471$
7%	$(100,000)(0.32268)$ $-3312\left(\frac{1-0.32268}{0.06542}\right)$ $= -2022.49$	$(1650,626,72)^2 (0.042567)$ $= 965,791,509$

c. (2 points) Calculate the $E[L_0]$.

Solution:

$$E[L_0] = (0.3)E[L_0 \mid i = 5\%] + (0.4)E[L_0 \mid i = 6\%] + (0.3)E[L_0 \mid i = 7\%]$$

$$= (0.03)(2644.04) + (0.4)(0) + (0.3)(-2022.49) = 186.46$$

d. (6 points) Calculate the $Var[L_0]$

Solution:

$$Var[L_0] = E(Var[L_0 \mid i]) + Var(E[L_0 \mid i])$$

$$= (0.3)(1,098,563,720) + (0.4)(1,034,064,471) + (0.3)(965,791,509)$$

$$+(0.3)(2644.04)^2 + (0.4)(0)^2 + (0.3)(-2022.49)^2 - (186.46)^2$$

$$= 1,032,932,357 + 3,289,655 = 1,036,222,012$$