STAT 475
Spring 2018
Quiz 3
April 10, 2018

1. (6 points) Sanchita who is (50) purchases a Type B universal life policy with an additional death benefit of 70,000. The cost of insurance for Sanchita's policy is 100% of the mortality rates in the Illustrated Life Table. Additionally, you are given that:

Policy Year	Annual Premium	Percent of Premium Charge	Annual Expense Charge	Annual Discount Rate for COI	Annual Credited Interest Rate
1	4000	25%	E	4%	6%

The account value at the end of one year is 2694.03.

Determine E.

Solution:

$$AV_1 = (AV_0 + P_1 - E_1 - COI_1)(1 + i_1^c)$$

$$COI_1 = ADB_1 \cdot v_q \cdot q_{50} = (70,000)(1.04)^{-1}(0.00592) = 398.46$$

$$2694.03 = (0 + 4000(0.75) - E - 398.46)(1.06)$$

$$E = 60$$

2. (8 points) Pratyush who is also (50) purchases an identical Type A universal life policy with an total death benefit of 100,000. The cost of insurance for Pratyush's policy is also 100% of the mortality rates in the Illustrated Life Table. Additionally, you are given that:

Policy Year	Annual Premium	Percent of Premium Charge	Annual Expense Charge	Annual Discount Rate for COI	Annual Credited Interest Rate	Account Value at End of Policy Year
1						8000
2	7000	10%	60	4%	5.5%	

Calculate the account value at the end of the second year.

Solution:

$$AV_2 = (AV_1 + P_2 - E_2 - COI_2)(1 + i_2^c)$$

$$COI_2 = ADB_2 \cdot v_q \cdot q_{51} = \{100,000 - [8000 + 7000(.9) - 60 - COI_2](1.055)\}(1.04)^{-1}(0.00642)$$

$$COI_2 = \frac{\{100,000 - [8000 + 7000(.9) - 60](1.055)\}(1.04)^{-1}(0.006420)}{1 - (1.055)(1.04)^{-1}(0.006420)} = 528.01$$

$$AV_2 = (8000 + (7000)(0.9) - 660 - 528.01)(1.055) = 14,466.15$$

3. (6 points) Aniruda is (60) and purchases a Type B Universal Life policy from Chenglin Life Insurance Company. Based on the assumptions below, Aniruda will have an account value of 23,000 at the end of 20 years:

Policy Year	Annual Premium	Percent of Premium Charge	Annual Expense Charge	Annual Discount Rate for COI	Annual Credited Interest Rate	Account Value at End of Policy Year
All Years	3000	10%	60	5%	5%	

Aniruda uses this information to determine that if he pays P each year (instead of 3000), he will have an account value of 0 (zero) after 20 years.

Determine P.

Solution:

$$AV_t = (AV_{t-1} + P_t(1-f) - EC_t - ADB_t v_q q_{x+t-1}^{COI})(1+i_t^c)$$

$$= (AV_{t-1} + P_t(1-0.1) - EC_t - \frac{ADB_t \cdot q_{x+t-1}^{COI}}{1.05})(1.05) = (AV_{t-1} + P_t(0.9) - EC_t)(1.05) - ADB_t \cdot q_{x+t-1}^{COI}$$

$$AV_1 = 0 + (0.9)P_1(1.05) - EC_1(1.05) - ADB_1 \cdot q_x^{COI}$$

$$AV_2 = \left[(0.9)P_1(1.05) - EC_1(1.05) - ADB_1 \cdot q_x^{COI} + P_2 - EC_2\right](1.05) - ADB_2 \cdot q_{x+1}^{COI} =$$

$$0.9\sum_{k=1}^{2} P_k(1.05)^{3-k} - \sum_{k=1}^{2} EC_k(1.05)^{3-k} - \sum_{k=1}^{2} ADB_k(1.05)^{2-k} q_{x+k-1}^{COI}$$

By Induction ==> $AV_t = 0.9\sum_{k=1}^{t} P_k(1.05)^{t+1-k} - \sum_{k=1}^{t} EC_k(1.05)^{t+1-k} - \sum_{k=1}^{t} ADB_k(1.05)^{t-k} q_{x+k-1}^{COI}$

Note that the second and third term are not a function of the premium so they will not be effected by the premium payment pattern.

Under premium payment pattern 1, we have:

(1) $AV_{20} = 23{,}000 = 0.9\sum_{k=1}^{20} 1000(1.05)^{20+1-k} - \sum_{k=1}^{20} EC_k(1.05)^{20+1-k} - \sum_{k=1}^{20} ADB_k(1.05)^{20-k} q_{x+k-1}^{COI}$

Under premium payment pattern 2, we have

(2) $AV_{20} = 0 = 0.9\sum_{k=1}^{20} (P)(1.05)^{20+1-k} - \sum_{k=1}^{20} EC_k(1.05)^{20+1-k} - \sum_{k=1}^{20} ADB_k(1.05)^{20-k} q_{x+k-1}^{COI}$

$$(1)-(2) = 23{,}000 = 0.9\sum_{k=1}^{20} (3000-P)(1.05)^{20+1-k} = (0.9)(3000-P)\ddot{s}_{\overline{20|}}$$

$$3000 - P = \frac{23{,}000}{0.9\left(\frac{1.05^{20}-1}{0.05/1.05}\right)} = 736.06 \implies P = 3000 - 736.06 = 2263.94$$