STAT 475
TEST 1
Spring 2018

February 15, 2018

1. Denis and Lucy are partners in H&R Consulting. Denis is (45) and Lucy is (35). They have purchased an insurance policy from Clark Life Insurance Company. The policy pays the following benefits:

   i. A death benefit of 500,000 paid at the end of the year of the second death; and

   ii. A reversionary annuity which pays 100,000 at the beginning of each year that Denis is alive but Lucy is dead.

   Premiums for this policy are paid annually as long as at least one of the partners is alive.

   You are given:

   i. Mortality follows the Illustrative Life Table.

   ii. Denis and Lucy are assumed to be independent lives.

   iii. $i = 0.06$.

   a. (6 points) The present value of benefits is 114,000 to the nearest 1000. Calculate it to the nearest 1.

   Solution:

   $$PVB = 500,000 A_{\overline{35:45}} + 100,000\left(\ddot{a}_{45} - \ddot{a}_{35:45}\right)$$

   $$= 500,000\left(A_{35} + A_{45} - A_{35:45}\right) + 100,000\left(\ddot{a}_{45} - \ddot{a}_{35:45}\right)$$

   $$= 500,000\left(0.20120 + 0.12872 - 0.24066\right) + 100,000\left(14.1121 - 13.4150\right)$$

   $$= 114,340$$

   b. (6 points) Calculate the annual premium for the policy using the equivalence principle.

   Solution:

   $$PVP = PVB \Longrightarrow P\ddot{a}_{\overline{35:45}} = 114,340 \Longrightarrow P\left(\ddot{a}_{35} + \ddot{a}_{45} - \ddot{a}_{35:45}\right) = 114,340$$

   $$P = \frac{114,340}{15.3926 + 14.1121 - 13.415} = 7,106.41$$

Clark Life Insurance Company offers a rider on the policy. The rider will pay the premium on the life insurance policy if only one of the partners is alive. This is equivalent to a survivor annuity that pays zero when both Denis and Lucy are alive but pays the premium amount when only Lucy or only Denis is alive.

c. (6 points) Calculate the Actuarial Present Value of the benefit under this rider.

Solution:

$$APV = a \cdot \ddot{a}_{35} + b \cdot \ddot{a}_{45} + c \cdot \ddot{a}_{35:45}$$

where a is the amount paid if only Lucy is alive and b is the amount paid if only Denis is alive. Additionally, a+b+c is the amount paid if they are both alive.

$$a = 7106.41; b = 7106.41; a+b+c = 0 ==> c = -2(7106.41)$$

$$APV = 7106.41\left(15.3926 + 14.1121 - 2(13.415)\right) = 19,007.51$$

You could also do this as $7106.41\left(\ddot{a}_{\overline{35:45}} - \ddot{a}_{35:45}\right)$

Premiums for this rider are paid annually, but only while both partners are alive.

d. (6 points) Calculate the annual premium for this rider.

Solution:

$$PVP = PVB ==> P\ddot{a}_{35:45} = 19,007.51 ==> P = 1416.88$$

Denis and Lucy decide to purchase the rider from Clark Life Insurance Company.

e. (6 points) Calculate the probability that the premium at the beginning of the 20th year will be paid by the rider.

Solution:

This is the probability that only one is alive at time 19. The 20th premium is paid at time 19.

$${}_{19}p_{\overline{35:45}} - {}_{19}p_{35:45} = {}_{19}p_{35} + {}_{19}p_{45} - {}_{19}p_{35} \cdot {}_{19}p_{45} - {}_{19}p_{35} \cdot {}_{19}p_{45}$$

$$= \frac{8,712,621}{9,420,657} + \frac{7,683,979}{9,164,051} - 2\left(\frac{8,712,621}{9,420,657}\right)\left(\frac{7,683,979}{9,164,051}\right) = 0.21239$$

2. Bryan is a participant in a defined benefit plan at Dai Actuarial Consultants. The retirement benefit is 2% of the three year final average salary for each year of service.

   You are given:

   i. Bryan was born on February 15, 1980.

   ii. Bryan was hired by Dai Actuarial Consultants on February 15, 2008.

   iii. Bryan has earned or will earned the following total salary during each timeframe listed in the table:

Date	Total Salary Earned
February 15, 2008 to February 15, 2015	210,000
February 15, 2015 to February 15, 2016	40,000
February 15, 2016 to February 15, 2017	44,000
February 15, 2017 to February 15, 2018	50,000
February 15, 2018 to February 15, 2019	60,000

   In completing your calculations, you should use the following assumptions:

   i. Bryan's salary will increase by 3% on each future February 15 beginning with February 15, 2019.

   ii. If Bryan remains employed by Dai, he will retire at exact age 65.

   iii. There is a 5% probability that Bryan will leave employment (for any reason) with Dai in each of the next ten years measured from February 15, 2018. After that, the probability that Bryan will leave employment is 2% each year until he is age 65.

   iv. The only benefit provided by the defined benefit plan is a retirement benefit.

   v. The interest rate prior to retirement is 4.5%.

   vi. The retirement benefit will be paid annually with the first payment at age 65. The benefit will be a single life annuity based on Bryan's life.

   vii. $\ddot{a}_{65} = 10$

   a. (6 points) Calculate Bryan's projected annual retirement benefit at age 65 using a Projected Unit Cost method.

      Solution:

$$Benefit = (\alpha)(FAS)(Number\ \ of\ \ Years)$$

$$= (0.02)\frac{60,000(1.03^{24} + 1.03^{25} + 1.03^{26})}{3}(37) = 92,990.82$$

February 20, 2018

b. (6 points) Calculate Bryan's projected replacement ratio.

Solution:

$$R = \frac{\text{First Year Retirement Benefit}}{\text{Salary in Final Year}} = \frac{92,990.82}{60,000(1.03)^{26}} = 0.7187$$

Dai hires Raza Retirement Consulting to complete a valuation on this plan as of February 15, 2018 using the Projected Unit Cost Method.

c. (6 points) Calculate the Accrued Benefit for Bryan using the Projected Unit Cost method.

Solution:

$$Benefit = (\alpha)(FAS)(Number\ of\ \ Years\ Accrued)$$

$$= (0.02)\frac{60,000(1.03^{24} + 1.03^{25} + 1.03^{26})}{3}(10) = 25,132.65$$

d. (6 points) Calculate the Accrued Actuarial Liability for Bryan as of February 15, 2018 using the Projected Unit Cost method.

Solution:

$$AAL = APV \text{ of Currently Accrued Benefit}$$

$$= (25,132.65)\ddot{a}_{65} \cdot v^{27} \cdot {}_{27}p_{38}^{(\tau)} = (25,132.65)(10)(1.045)^{-27}(0.95)^{10}(0.98)^{17}$$

$$= 32,522.04$$

e. (6 points) Calculate the Normal Contribution on February 15, 2018 for the year ended February 15, 2019 using the Projected Unit Cost Method.

Solution:

$$ {}_{10}V + C_{10} = EPVofBenefitsPaid + v \cdot {}_1 p_{38}^{(\tau)} \cdot {}_{t+1}V $$

$$ {}_{10}V = 32,522.04 $$

$$ EPVofBenefitsPaid = 0 $$

$$ (1.045)^{-1} $$

$$ {}_1 p_{38}^{(\tau)} = 0.95 $$

$$ {}_{t+1}V $$

$$ = \left((0.02) \frac{60,000(1.03^{24} + 1.03^{25} + 1.03^{26})}{3} (11) \right) \ddot{a}_{65} \cdot v^{26} \cdot {}_{26} p_{39}^{(\tau)} = $$

$$ \left((0.02) \frac{60,000(1.03^{24} + 1.03^{25} + 1.03^{26})}{3} (11) \right) (10)(1.045)^{-26} (0.95)^9 (0.98)^{17} $$

$$ = 39,351.67 $$

$$ C_{10} = (39,351.67)(1.045)^{-1}(0.95) - 32,522.04 = 3252.21 $$

3. Jain Life Insurance Company sells a two year term life insurance policy to (80) that pays 100,000 at the end of the year of death if death is from natural causes and pays 200,000 at the end of the year of death if death is from accidental causes.

   Sanchita prices this policy using a multiple decrement table where decrement (1) is death from natural causes, decrement (2) is death from accidental causes, and decrement (3) is lapse.

   If the policy lapses, no benefits are paid.

   Premiums for this policy are paid annually as long as the policy is in force.

   You are given:

   i. The following multiple decrement table:

x	$q_x^{(1)}$	$q_x^{(2)}$	$q_x^{(3)}$
80	0.08	0.02	0.20
81	0.12	0.04	0.00

   ii. $v = 0.92$

   a. (6 points) The Actuarial Present Value of Benefits is 22,900 to the nearest 100. Calculate it to the nearest 1.

   Solution:

x	$l_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$	$d_x^{(3)}$
80	10,000	800	200	2000
81	7000	840	280	0
82	5880			

$$PVB = 100,000\left(\frac{800v + 840v^2}{10,000}\right) + 200,000\left(\frac{200v + 280v^2}{10,000}\right) = 22,889.60$$

   b. (6 points) Calculate the annual net benefit premium for this policy using the equivalence principle.

   Solution:

$$PVP = PVB \Longrightarrow P\left(\frac{10,000 + 7000v}{10,000}\right) = 22,889.60 \Longrightarrow P = 13,923.11$$

c. (18 points) Let L_0 be the present value loss at issue random variable for this policy. In order to calculate the $Var[L_0]$, you must use first principles. There are six possible values of the random variable. Complete the table below by describing each case, the value of the random variable in that case, and probability that the case will happen. You do NOT need to calculate the variance. Show your work.

Case	Description of Case	Value of L_0	Probability
1	Die Year 1 Natural Causes	$100,000v - 13,923.11$ $= 78,076.89$	This column comes from the table in Part a. 0.08
2	Die Year 1 Accidental Causes	$200,000v - 13,923.11$ $= 170,076.89$	0.02
3	Lapse Year 1	$0 - 13,923.11$ $= -13,923.11$	0.20
4	Die Year 2 Natural Causes	$100,000v^2 - 13,923.11(1+v)$ $= 57,907.63$	0.084
5	Die Year 2 Accidental Causes	$200,000v^2 - 13,923.11(1+v)$ $= 142,547.63$	0.028

6	Live 2 Years	$0-13,923.11(1+v)$ $=-26,732.37$	0.588

4. (10 points) You are given the following independent single decrement rates: $q'^{(1)}_{50} = 0.10$ and $q'^{(2)}_{50} = 0.20$

   Decrement (1) occurs uniformly throughout the year. Decrement (2) occurs three quarters of the way throughout the year.

   Complete the following multiple decrement table. Be sure to show your work.

Age	$l_x^{(\tau)}$	$d_x^{(1)}$	$d_x^{(2)}$
50	10,000		

Solution:

$$l_{x+1}^{(\tau)} = l_x^{(\tau)} \cdot p'^{(1)}_x \cdot p'^{(2)}_x = (10,000)(0.90)(0.80) = 7200$$

Therefore, we know that $d_x^{(1)} + d_x^{(2)} = 10,000 - 7200 = 2800$.

If there were no other decrements, there would be (10,000)(0.1) = 1000 decrement (1). These are uniformly distributed so there are 750 in the first 3 quarters of the year and 250 in the last quarter of the year. Since decrement (2) occurs 3 quarters of the way throughout the year, the 750 decrement (1) occur because there are no other decrements during this time. At that point, there are 10,000-750=9250 that are exposed to decrement (2). Of these, 20% decrement at that point in time (9250)(0.2)=1850. If decrement (2) is 1850, decrement (1) must be 2800-1850=950.