STAT 475
Test 2
Spring 2018
March 22, 2018

1. You are given the following mortality table:

x	q_x
90	0.10
91	0.20
92	0.30
93	0.50
94	0.75
95	1.00

You are given the following term structure of interest rates:

t	y_t
1	4.00%
2	5.00%
3	6.00%
4	7.00%
5	8.00%
6	9.00%

The Ribordy Life Insurance Company sells a fully discrete four year term policy to (90). The policy provides a death benefit of 250,000.

a. (6 points) The annual net premium for this policy is 54,000 to the nearest 1000. Calculate the annual net premium to the nearest 1.

Solution:

$$l_{90} = 1000; l_{91} = 900; l_{92} = 720; l_{93} = 504; l_{94} = 252$$

$$PVP = PVB$$

$$P\left[1000 + 900v(1) + 720v(2) + 504v(3)\right]$$
$$= (250,000)\left[(1000-900)v(1) + (900-720)v(2) + (720-504)v(3) + (504-252)v(4)\right]$$

$$P\left[1000 + 900(1.04)^{-1} + 720(1.05)^{-2} + 504(1.06)^{-3}\right]$$
$$= (250,000)\left[100(1.04)^{-1} + 180(1.05)^{-2} + 216(1.06)^{-3} + 252(1.07)^{-4}\right]$$

$$P = 53,799.25$$

b. (14 points) Calculate the reserve for this policy at the end of the second year.

Solution:

$${}_0V = 0$$

$${}_1V = \frac{(0 + 53,799.25)(1.04) - (250,000)(0.1)}{1-0.1} = 34,390.2456$$

$$f_{[1,2]} = \frac{(1.05)^2}{1.04} - 1 = 0.060096154$$

$${}_2V = \frac{(34,390.2456 + 53,799.25)(1.060096154) - (250,000)(0.2)}{1-0.2} = 54,361.68$$

2. The Thomas Life Insurance Company sells a fully discrete whole life insurance policy to (50). The policy pays a death benefit of 10,000 at the end of the year of death. The policy has annual premiums paid at the beginning of each policy year.

   You are given:

   i. Mortality follows the Illustrative Life Table.

   ii. Interest rates are uncertain but are distributed as follows:

      1. 5.5% with a probability of 80%

      2. 8.0% with a probability of 20%

   iii. Net annual premiums are determined using 6% which is the expected value of the interest rate.

   iv. The following table of values:

i	d	A_{50}	${}^2A_{50}$
5.5%	0.05213	0.27422	0.10806
6.0%	0.05660	0.24905	0.09476
8.0%	0.07407	0.17498	0.06089

a. (8 points) The net annual premium is 190 to the nearest 10. Calculate it to the nearest 0.01.

Solution:

$$PVP = PVB$$

$$P\ddot{a}_{50} = (10,000)A_{50}$$

$$P = \frac{(10,000)(0.24905)}{13.2668} = 187.71$$

The loss at issue random variable for this policy is L_0.

b. (10 points) The $E[L_0] = 30$ to the nearest 10. Calculate the $E[L_0]$ to the nearest 0.05.

Solution:

$$E[L_0 \mid i = 0.055] = (10,000)(0.27422) - 187.71\left(\frac{1-0.27422}{0.05213}\right) = 128.7798985$$

$$E[L_0 \mid i = 0.08] = (10,000)(0.17498) - 187.71\left(\frac{1-0.17498}{0.07407}\right) = -341.0076858$$

$$E[L_0] = (128.7798985)(0.8) + (-341.0076858)(0.2) = 34.82$$

c. (15 points) The $Var[L_0] = 6,000,000$ to the nearest 1,000,000. Calculate the $Var[L_0]$ to the nearest 100.

Solution:

$$Var[L_0 \mid i = 0.055] = \left(10,000 + \frac{187.71}{0.05213}\right)^2 [0.10806 - (0.27422)^2] = 6,079,166.809$$

$$Var[L_0 \mid i = 0.08] = \left(10,000 + \frac{187.71}{0.07407}\right)^2 [0.06089 - (0.17498)^2] = 4,755,956.539$$

$$Var[L_0] = E[Var(L_0)] + Var[E(L_0)]$$

$$= \{(6,079,166.809)(0.8) + (4,755,956.539)(0.2)\}$$
$$+ \{(128.7798985)^2(0.8) + (-341.0076858)^2(0.2) - (34.82)^2\}$$

$$= 5,849,837$$

d. (7 points) Thomas sells 1000 of these whole life policies to independent lives. Using the normal approximation, determine the probability that Thomas will incur a loss at issue in excess of 100,000.

Solution:

$$\Pr\left[L_0 > 100,000\right] = \Pr\left[Z > \frac{100,000-(1000)(34.82)}{\sqrt{(1000)(5,849,837)}}\right]$$

$$= \Pr\left[Z > 0.8522\right] = \Pr\left[Z > 0.85\right] = 1 - \Pr\left[Z < 0.85\right]$$

$$= 1 - 0.8023 = 0.1977$$

3. For a three year term insurance policy on (x), you are given the follow profits (profit vector) and premiums for each policy that is in force at the start of the policy year. You are also given the decrements for each year.

Time (t)	Profit Vector	Premium at t-1	q_{x+t-1}	Withdrawal Rate at t
0	-1000			
1	450	2400	0.05	0.20
2	$\Pr_2$	2400	0.08	0.10
3	900	2400	0.12	0.00

a. (8 points) This policy has an Internal Rate of Return of 25%. Calculate $\Pr_2$.

Solution:

$$\pi_0 = -1000; \pi_1 = 450; \pi_2 = (0.8)(0.95)\Pr_2 = 0.76\Pr_2$$
$$\pi_3 = (0.8)(0.95)(0.9)(0.92)(900) = 566.352$$

$$-1000 + 450(1.25)^{-1} + 0.76\Pr_2(1.25)^{-2} + 566.352(1.25)^{-3} = 0$$

$$\Pr_2 = \frac{1000 - 450(1.25)^{-1} - 566.352(1.25)^{-3}}{0.76(1.25)^{-2}} = 719.63$$

b. (6 points) Calculate the Net Present Value of the profits using a 15% interest rate.

Solution:

$$NPV = -1000 + 450(1.15)^{-1} + (719.63)(0.76)(1.15)^{-2} + (566.325)(1.15)^{-3}$$

$$= 177.24$$

c. (6 points) Calculate the profit margin for this policy using a 15% interest rate.

Solution:

$$PM = \frac{NPV}{PVP} = \frac{177.24}{2400 + (2400)(0.8)(0.95)(1.15)^{-1} + 2400(0.8)(0.95)(0.9)(0.92)(1.15))^{-2}}$$

$$= 0.03456$$

4. (20 points) A fully discrete participating whole life on (60) provides a death benefit of 1,000,000. You are given:

   i. The gross premium paid annually is 37,000.
   ii. The reserve at the end of the 19^{th} year is 448,000.
   iii. The reserve at the end of the 20^{th} year is 470,000.
   iv. The reserve at the end of the 21^{th} year is 493,000.
   v. Cash values are 95% of the reserves
   vi. Mortality is equal to 90% the Illustrative Life Table.
   vii. Withdrawals are 20% in the first year, 10% in the second year, and 5% all years thereafter.
   viii. The interest rate earned is 7% in all years.
   ix. Issue expenses are 40% of premium plus 500 per policy.
   x. Maintenance expenses are 6% of premium plus 50 per policy in all years including the first year.
   xi. The policy pays a dividend of 75% of the profit.

   Calculate the dividend in the 21^{th} year.

   Solution:

$${}_{20}V = 470{,}000$$
$$P_{20} = 37{,}000$$
$$Exp_{20} = (0.06)(37{,}000) + 50 = 2270$$
$$Int_{20} = (470{,}000 + 37{,}000 - 2270)(0.07) = 35{,}331.10$$
$$DB_{20} = (1{,}000{,}000)(0.9)(0.0803) = 72{,}270$$
$$With_{20} = (0.95)(493{,}000)(1-(0.9)(0.0803))(0.05) = 21{,}725.12$$
$${}_{21}V \cdot p_{80}^{(\tau)} = (493{,}000)((1-(0.9)(0.0803))(1-0.05) = 434{,}502.35$$

$$\Pr_{21} = 470{,}000 + 37{,}000 - 2270 + 35{,}331.10 - 72{,}270 - 21{,}725.12 + 434{,}502.35$$

$$= 11{,}563.63$$

$$Div_{21} = (0.75)(11{,}563.63) = 8672.72$$