Quiz 1

Fall 2008

(1) $E(X) = 8 = \dfrac{\alpha\theta}{\alpha-1} = \dfrac{4\alpha}{\alpha-1}$

$\Rightarrow 8(\alpha-1) = 4\alpha \Rightarrow 8\alpha - 8 = 4\alpha \Rightarrow \alpha = 2$

$S(x) = 1 - F(x) = \left(\dfrac{\theta}{x}\right)^{\alpha}$

$S(6) = \left(\dfrac{4}{6}\right)^{2} = 4/9$

(2) Mode $= \theta(\alpha-1) = 3000 \Rightarrow \theta(3) = 3000 \Rightarrow \theta = 1000$

$E[X] = \theta\alpha = 1000(4) = 4000$

$E[X] = \theta^2(\alpha+1)(\alpha) = (1000)^2(5)(4) = 20{,}000{,}000$

$Var(X) = E(X^2) - (E(X))^2 = 4{,}000{,}000$

$\sigma = \sqrt{Var(X)} = 2000$

Quiz 2

Fall 2008

① $E(X) = 100(.3) + 400(.1) + 900(.1) = 160$

$E(X^2) = (100^2)(.3) + (400)^2(.1) + (900)^2(.1) = 100{,}000$

$Var(X) = 100{,}000 - (160)^2 = 74{,}400$

$S_{50} = X_1 + X_2 + \cdots + X_n$

$E(S_{50}) = (50)(160) = 8000$

$Var(S_{50}) = (50)(Var(X)) = 3{,}720{,}000$

$$\left(\frac{x - 8000}{\sqrt{3{,}720{,}000}}\right) = 1.645$$

$x = 11{,}172$

② $p_1 = e^{-\lambda}(\lambda)$

$f_\Lambda(\lambda) = \frac{e^{-\lambda/\theta}}{\theta} = e^{-\lambda}$ GAMMA with $\alpha = 1$ is exponential or by using GAMMA with $\alpha = 1$ & $\theta = 1$

Probability of one claim $= \int_0^\infty e^{-\lambda}(\lambda)\, e^{-\lambda}\, d\lambda$

$$= \int_0^\infty \lambda e^{-2\lambda}\, d\lambda = \frac{e^{-2\lambda}}{4}(-2\lambda - 1)\Big|_0^\infty$$

$$= 0 - \frac{e^{-0}}{4}(-0-1) = 1/4$$

Bonus Probability of zero claims $= \int_0^\infty e^{-\lambda} e^{-\lambda}\, d\lambda = 1/2$

Probability of one claim $= 1/4$

Probability of two claims $= \int_0^\infty 1/2\, e^{-\lambda}\lambda^2\, e^{-\lambda}\, d\lambda = 1/8$

BY INDUCTION $= (1/2)^{n+1}$

OR

$$p_n = \int_0^\infty \frac{1}{n!} \lambda^n e^{-2\lambda} \, d\lambda$$

$$= \left(\frac{1}{n!}\right)\left(\frac{1}{-2} \lambda^n e^{-2\lambda} \Big|_0^\infty - \frac{n}{-2} \int_0^\infty \lambda^{n-1} e^{-2\lambda} d\lambda \right)$$

$$= 1/n! \left(0 + \frac{n}{2} \int_0^\infty \lambda^{n-1} e^{-2\lambda} d\lambda \right)$$

$$= 1/2 \int_0^\infty \frac{1}{(n-1)!} \lambda^{n-1} e^{-2\lambda} dx$$

$$= 1/2 \cdot p_{n-1}$$

$$p_0 = 1/2$$

$$p_n = (1/2)^{n+1}$$

Quiz 3
Fall 2008

(1)

$$E(Y^P) = \frac{E(Y^L)}{1-F(1)} = \frac{E(X) - E(X\wedge 1)}{1-F(1)}$$

$$p_0 = (1+1)^{-2} = \frac{1}{4} \qquad p_1 = \frac{(2)(1)^1}{1!(2)^{2+1}} = \frac{1}{4}$$

$$1-F(1) = 1 - 1/4 - 1/4 = 1/2 \qquad E(X) = \alpha\beta = 2$$

$$E(X\wedge 1) = \sum_0^1 x\, p_x + (1)(1-F(1))$$

$$= (0)(1/4) + (1)(1/4) + (1)(1/2) = 3/4$$

$$E(Y^P) = \frac{2 - 3/4}{1 - 1/2} = 2.5$$

(2) $X \sim$ Poisson ($\lambda = 8/15$ mins) $\Rightarrow X \sim$ Poisson ($\lambda = 32$/hour)

20% are men $\sim$ Poisson ($\lambda = 6.4$/hour)

$$P(X<4) = P(0) + P(1) + P(2) + P(3)$$

$$= e^{-6.4}\left[1 + 6.4 + \frac{(6.4)^2}{2!} + \frac{(6.4)^3}{3!}\right]$$

$$= 0.1189$$

Math 490C
Fall 2008
Quiz 4

1. The number of claims in a year for a Hospital Indemnity Policy have the following distribution:

N	p_n
0	.4
1	.3
2	.2
3	.1

The amount of each claim has the following distribution:

X	f(x)
1000	0.75
2000	0.25

A stop loss policy is purchased to cover all claims in excess of an aggregate amount equal to 120% of the expected aggregate claims.

Calculate the net stop loss premium.

$E(N) = (0)(.4) + (1)(.3) + (2)(.2) + (3)(.1) = 1.0$

$E(X) = (1000)(.75) + (2000)(.25) = 1250$

$E(S) = E(N)E(S) = 1250$

$d = (1.2)(1250) = 1500$

Net Stop Loss Premium $= E(S) - E(S \wedge 1500)$

$f_S(0) = .4$

$f_S(1000) = (.3)(.75) = .225$

$E(S \wedge 1500) = (0)(.4) + (1000)(.225) + 1500(1 - .4 - .225)$

$= 0 + 225 + 562.50 = 787.50$

Net Stop Loss Premium $= 1250 - 787.50 = 462.50$

2. An automobile insurer has 1000 cars covered during **2008**. The number of automobile claims for each car follows a Poisson distribution with $\lambda = 0.5$. Each claim during **2007** distributed exponentially with a mean of 5000. Claims in **2008** are subject to uniform inflation of 10%. Assume that the number of claims and the amount of the loss are independent and identically distributed.

Using the normal distribution as an approximating distribution of aggregate losses, calculate the probability that losses will exceed 3 million.

$E(N) = \lambda = 0.50$ $\qquad \theta^{2008} = (1.1)(\theta^{2007}) = 5500$

$E(X) = \theta^{2008} = 5500$

$E(S) = (.5)(5500) = 2750$

$Var(N) = \lambda = 0.50$

$Var(X) = (\theta^{2008})^2 = (5500)^2$

$E(\text{total losses}) = 1000\, E(S) = 2{,}750{,}000$

$Var(\text{total losses}) = 1000\, Var(S) = (1000)(5500)^2$

$Pr(\text{Total losses} > 3{,}000{,}000) =$

$$Pr\left(Z > \frac{3{,}000{,}000 - 2{,}750{,}000}{\sqrt{1000(5500)^2}}\right)$$

$= Pr(Z > 1.44) = 1 - \Phi(1.44)$

$= 1 - .9251 = .0749$

QUIZ 5 2008

① $A = F(1100) - F(900) = \left[1 - \left(\frac{3000}{4100}\right)^4\right] - \left[1 - \left(\frac{3000}{3900}\right)^4\right]$

$$= \left(\frac{3000}{3900}\right)^4 - \left(\frac{3000}{4100}\right)^4 = 0.06347935$$

$$B = \frac{2 \cdot E(X \wedge 1000) - E(X \wedge 800) - E(X \wedge 1200)}{200}$$

$$= \frac{2\,\frac{3000}{3}\left[1 - \left(\frac{3000}{4000}\right)^3\right] - \frac{3000}{3}\left[1 - \left(\frac{3000}{3800}\right)^3\right] - \frac{3000}{3}\left[1 - \left(\frac{3000}{4200}\right)^3\right]}{200}$$

$$= 0.06367861$$

$$100000\,(B - A) = 19.926$$

② $E(N) = 15$ we want $E(N^P) \le 5$

N is distributed Poisson with $\lambda = 15$

Then N^P is distributed Poisson with $\lambda = 15v$

where $v = 1 - F_X(d)$

so $\lambda^{N^P} = \lambda^N (1 - F_X(d))$

$$5 = 15\left(1 - \left[1 - e^{-\left(\frac{d}{500}\right)^2}\right]\right) = 15\,e^{-\left(\frac{d}{500}\right)^2}$$

$$e^{-\left(\frac{d}{500}\right)^2} = 1/3 \Rightarrow \left(\frac{d}{500}\right)^2 = -\ln 1/3$$

$$d = 500\sqrt{-\ln 1/3} = 524.07$$

Quiz 6 Fall 2008

(1)

$$A = f(3000) = F_X(3150) - F_X(2850)$$

$$= \left[1 - \left(\frac{10,000}{13,150}\right)^4\right] - \left[1 - \left(\frac{10000}{12,850}\right)^4\right] = .03234011$$

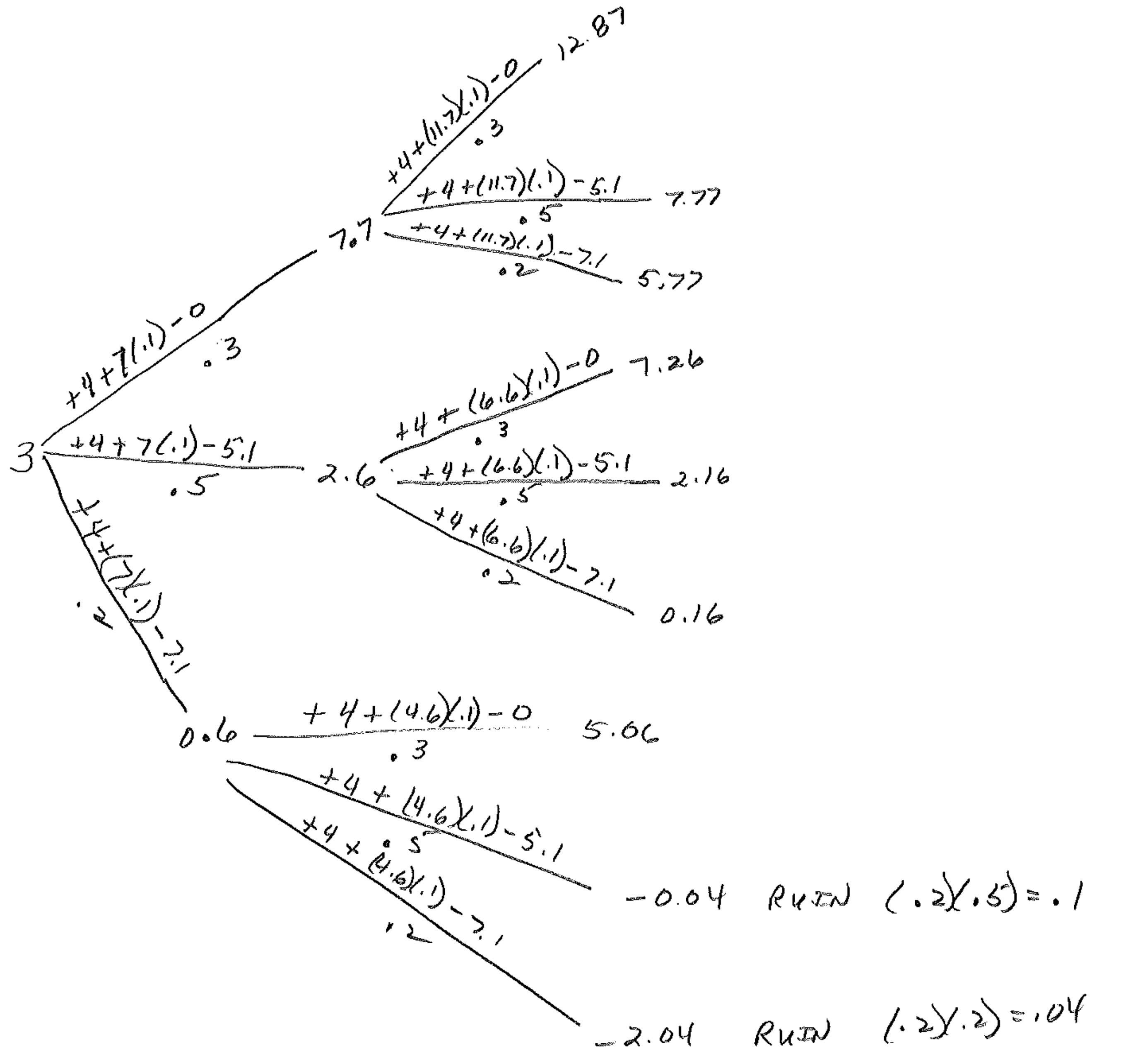
$$B = \frac{2E(X \wedge 3000) - E(X \wedge 2700) - E(X \wedge 3300)}{300}$$

$$= \frac{1}{300}\left\{2\left(\frac{10000}{3}\right)\left[1 - \left(\frac{10000}{13,000}\right)^3\right] - \frac{10000}{3}\left[1 - \left(\frac{10000}{12,700}\right)^3\right] - \frac{10000}{3}\left[1 - \left(\frac{10,000}{13,300}\right)^3\right]\right\}$$

$$= 0.0323625 61$$

$$100,000(B - A) = 2.15492$$

(2)



TOTAL .14

Probability of survival =

1 - RUIN = 0.86

Math 490C
Fall 2008
Quiz 7

1. A sample of 16 is taken from a Pareto distribution with parameters θ and α = 4.

The mean of the distribution is estimated using the sample mean as the estimator.

The Mean Square Error of the estimator = 200.

Calculate θ.

$$MSE = Var(\hat{\theta} \mid \theta) + [bias_{\hat{\theta}}(\theta)]^2$$

$\bar{X}$ is an unbiased estimator

so $bias_{\hat{\theta}}(\theta) = 0$

$$Var(\bar{X}) = Var\left(\frac{X_1 + X_2 + \cdots + X_n}{n}\right) =$$

$$\frac{1}{n^2} Var(X_1 + X_2 + \cdots + X_n) = \frac{1}{n^2}\left[Var(X_1) + Var(X_2) + \cdots + Var(X_n)\right]$$

$$= \frac{1}{n^2} \cdot n\, Var(X) = \frac{Var(X)}{n} = \frac{Var(X)}{16} \text{ since } n = 16$$

Under Pareto, $Var(X) = \dfrac{\theta^2(\alpha)}{(\alpha-1)^2(\alpha-2)} = \dfrac{\theta^2(4)}{(9)(2)}$

$$MSE = 200 = \frac{Var(X)}{16} = \frac{\theta^2(4)}{(16)(18)}$$

$$\therefore \theta = \sqrt{\frac{200(16)(18)}{4}} = 120$$

2. A sample of n is taken from a uniform distribution on $(0, \theta)$.

θ is estimated using the estimator:

$$\hat{\theta} = (2n\bar{X})/(n-1)$$

Calculate the bias in this estimator.

$$\text{bias} = E(\hat{\theta}|\theta) - \theta$$

$$E(\hat{\theta}|\theta) = E\left[\frac{2n}{n-1}\bar{X}\right] =$$

$$\frac{2n}{n-1}E(\bar{X}) = \frac{2n}{n-1}E\left(\frac{X_1 + X_2 + \dots + X_n}{n}\right) = \frac{2n}{(n)(n-1)}E(X_1 + \dots + X_n)$$

$$= \frac{2n}{(n-1)(n)}\left[\frac{\theta}{2} + \frac{\theta}{2} + \dots + \frac{\theta}{2}\right] = \frac{2n}{n(n-1)}\left[n \cdot \frac{\theta}{2}\right]$$

$$= \frac{2}{n-1} \cdot n \cdot \frac{\theta}{2} = \frac{n}{n-1}\theta$$

$$\text{bias} = \frac{n}{n-1}\theta - \theta = \frac{n-(n-1)}{n-1}\theta = \frac{\theta}{n-1}$$

Quiz 8

(1)

j	time	s_j	r_j
1	0.3	1	14
2	0.5	2	12
3	1.0	2	11

$$\hat{H}_{20}(1.0) = \frac{1}{14} + \frac{2}{12} + \frac{2}{11} = .41991342$$

$$S_{20}(1.0) = e^{-\hat{H}_{20}(1.0)} = .6571$$

(2)

j	time	s_j	r_j
1	0.3	1	14
2	0.5	2	12
3	1.0	2	11
4	2.5	1	11

Note: We do not use since occurs at 2.5 & we want $S_{20}(2.0)$

$$S_{20}(2.0) = \left(\frac{14-1}{14}\right)\left(\frac{12-2}{12}\right)\left(\frac{11-2}{11}\right) = 0.6331$$

QUIZ 9 FALL 2008

(1)

y_i	s_i	r_i
0.3	1	15 at start − 1 lapse at .2 = 14
0.5	2	14 − death at 0.3 − 1 lapse at 0.4 = 12
1.0	2	12 + 2 enter at 0.5 − 2 deaths at 0.5 − 1 lapse at 0.5 = 11

$$\mathrm{Var}(\hat{H}(1.0)) = \sum_{i=1}^{3} \frac{s_i}{(r_i)^2} = \frac{1}{(14)^2} + \frac{2}{(12)^2} + \frac{2}{(11)^2} = 0.03552$$

(2)

y_i	p_{y_i}	$K_{y_i}(x)$	
8	$\frac{1}{10}$	1	← since 21 is greater than 8+6
12	$\frac{1}{10}$	1	← since 21 is greater than 12+6
15	$2/10$	1	← since 21 is = 15+6
18	$1/10$	$\frac{21-18+6}{(6)(2)} = \frac{9}{12}$	
24	$1/10$	$\frac{21-24+6}{(6)(2)} = 3/4$	

we can ignore the rest because 28−6=22 which is higher than 21.

$$\hat{F}(21) = \left(\frac{1}{10}\right)(1) + \left(\frac{1}{10}\right)(1) + \left(\frac{2}{10}\right)(1) + \frac{1}{10}\left(\frac{9}{12}\right) + \frac{1}{10}\left(3/12\right)$$

$$= \frac{4}{10} + \frac{9}{120} + \frac{3}{120} = .5$$

Quiz 10 2008

(1)

x	$F_5(x^-)$	$F_5(x)$	$F^*(x)$ ← $1-e^{-\frac{x}{600}}$	$\lvert\max\rvert$
100	0	.2	.1535	.1535
200	.2	.4	.2835	.1165
400	.4	.6	.4866	.1134
800	.6	.8	.7364	.1364
1500	.8	1.0	.9179	.1179

MAX .1535 is K-S Test stat

$$\text{MLE} = \bar{X} = \hat{\theta} = \frac{100+200+400+800+1500}{5} = 600$$

$$\text{critical value} = \frac{1.36}{\sqrt{5}} = .6082$$

Since $.1535 < .6082$ we do not reject hypothesis

(2) $$L(\theta) = f(200)\cdot\left[1 - F(1000)\right] = \frac{\alpha\,(2500)^\alpha}{(2700)^{\alpha+1}} \cdot \left(\frac{2500}{3500}\right)^\alpha$$

$$= \frac{\alpha\left(\frac{25}{27}\right)^\alpha}{2700} \cdot \left(\frac{5}{7}\right)^\alpha = \frac{\alpha\left(\frac{25}{27}\cdot\frac{5}{7}\right)^\alpha}{2700}$$

$$l(\theta) = \ln\alpha + \alpha\ln\left(\frac{25}{27}\cdot\frac{5}{7}\right) - \ln 2700$$

$$l'(\theta) = \frac{1}{\alpha} + \ln\left(\frac{25}{27}\cdot\frac{5}{7}\right) - 0 = 0$$

$$\alpha = \frac{1}{-\ln\left(\frac{25}{27}\cdot\frac{5}{7}\right)} = 2.41877$$

QUIZ 10

(1)

X	$F_5(x^-)$	$F_5(x)$	$F^*(x)$	$\lvert MAX \rvert$
100	0	.2	.1535	.1535
200	.2	.4	.2835	.1165
400	.4	.6	.4866	.1134
800	.6	.8	.7364	.1364
1500	.8	1.0	.9179	.1179

$F^*(x) = 1 - e^{-\frac{x}{600}}$

$$MLE = \bar{X} = \hat{\theta} = \frac{100+200+400+800+1500}{5} = 600$$

MAX .1535 is K-S Test stat

$$\text{critical value} = \frac{1.36}{\sqrt{5}} = .6082$$

Since $.1535 < .6082$ we do not reject hypothesis

(2) $$L(\theta) = f(200)\cdot\left[1 - F(1000)\right] = \frac{\alpha\,(2500)^{\alpha}}{(2700)^{\alpha+1}}\cdot\left(\frac{2500}{3500}\right)^{\alpha}$$

$$= \frac{\alpha\left(\frac{25}{27}\right)^{\alpha}}{2700}\cdot\left(\frac{5}{7}\right)^{\alpha} = \frac{\alpha\left(\frac{25}{27}\cdot\frac{5}{7}\right)^{\alpha}}{2700}$$

$$l(\theta) = \ln\alpha + \alpha\ln\left(\frac{25}{27}\cdot\frac{5}{7}\right) - \ln 2700$$

$$l'(\theta) = \frac{1}{\alpha} + \ln\left(\frac{25}{27}\cdot\frac{5}{7}\right) - 0 = 0$$

$$\alpha = \frac{1}{-\ln\left(\frac{25}{27}\cdot\frac{5}{7}\right)} = 2.41877$$