STAT 490 C

TEST 1

① Y = Hong Kong Dollars $\quad Y = 7.8X$

$$Var(Y) = Var(7.8X) = (7.8)^2 \, Var(X) = (7.8)^2(10) = 608.40$$

or θ is a scale variable. with $\theta^{HKD} = 7.8\,\theta^{USD}$

$$Var = \alpha\theta^2 \Rightarrow \alpha(7.8\theta)^2 = (7.8)^2(\alpha\theta^2) = (7.8)^2(10) = 608.40$$

② $E(X) = \dfrac{\theta}{\alpha-1} = \dfrac{1000}{3-1} = 500 \qquad E(Y^P) = E(X)(1.3) = 650$

$$E(Y^P) = \frac{E(X) - E(X\wedge d)}{1-F(d)} + d$$

$$= \frac{\frac{\theta}{\alpha-1} - \frac{\theta}{\alpha-1}\left[1-\left(\frac{\theta}{\theta+d}\right)^{\alpha-1}\right]}{1-\left[1-\left(\frac{\theta}{\theta+d}\right)^{\alpha}\right]} + d$$

$$= \frac{\frac{\theta}{\alpha-1}\left(\frac{\theta}{\theta+d}\right)^{\alpha-1}}{\left(\frac{\theta}{\theta+d}\right)^{\alpha}} + d = \frac{\theta}{\alpha-1}\cdot\frac{\theta+d}{\theta} + d$$

$$= 500\cdot\frac{1000+d}{1000} + d = 650$$

$$= 500 + \tfrac{1}{2}d + d = 650$$

$$1.5d = 150$$

$$\Rightarrow d = 100$$

③ $E(Y) = (100)(.4) + 200(.3) + 300(.2) + 400(.1) = 200$

$Var(Y) = (100-200)^2(.4) + (0)(.3) + (300-200)^2(.2) + (400-200)^2(.1) =$ 10,000

$f(x) = F'(x) = -(-4)(100)^4 x^{-5} = 4(100)^4(x^{-5})$

$E(X) = \int_{100}^{\infty} x f(x)\,dx = \int_{100}^{\infty} 4(100)^4 x^{-4}\,dx =$

$4(100)^4 \left.\frac{x^{-3}}{-3}\right]_{100}^{\infty} = \frac{4(100)^4}{3(100)^3} = 133.3\bar{3}$

$E(X^2) = \int_{100}^{\infty} x^2 f(x)\,dx = \int_{100}^{\infty} 4(100)^4 x^{-3}\,dx$

$= 4(100)^4 \left.\frac{x^{-2}}{-2}\right]_{100}^{\infty} = \frac{4(100)^4}{2(100)^2} = 20000$

$Var(X) = 20{,}000 - (133.3\bar{3})^2 = 2222.22$

$Var(Y) - Var(X) = 10{,}000 - 2222.22 = 7777.78$

Alternatively

$F(x) = 1 - \left(\frac{100}{x}\right)^4$ is single parameter Pareto with $\theta = 100$ and $\alpha = 4$

so $E(X) = \frac{\alpha\theta}{\alpha - 1} = \frac{400}{3} = 133.33$

$E(X^2) = \frac{\alpha\theta^2}{\alpha - 2} = \frac{(4)(100^2)}{2} = 20{,}000$

④ $E(N) = \beta = 2 \qquad Var(N) = \beta(1+\beta) = (2)(3) = 6$

$E(X) = \alpha\theta = (1)(100) = 100 \qquad Var(X) = \alpha\theta^2 = (1)(100)(100) = 10{,}000$

$E(S) = E(N)E(X) = (2)(100) = 200 \qquad E(\text{TOTAL LOSSES}) = (1000)(200) = 200{,}000$

$$Var(S) = E(N)\,Var(X) + Var(N)\,[E(X)]^2$$
$$= (2)(100^2) + (6)(100^2) = 8(100^2)$$

$$Var(\text{TOTAL LOSSES}) = 1000\,Var(S) = (1000)(8)(100^2)$$

$$Pr(\text{TOTAL LOSSES} < X) = .85$$

$$\Rightarrow \quad \frac{X - 200{,}000}{\sqrt{(1000)(8)(100^2)}} = 1.036$$

$$\Rightarrow X = 209{,}266$$

⑤ $\overset{\text{UNMOD}}{Var}(N) = 20 = \beta(1+\beta) \implies \beta = 4$

$$\overset{\text{ZERO MOD}}{Var}(N) = 20.25 = (1-p_0^m)\left(Var(N)^{\text{ZERO TRUNCATED}}\right) + (p_0^m)(1-p_0^m)\left(E(N)^{\text{ZERO TRUNCATED}}\right)^2$$

$$20.25 = (1-p_0^m)(20) + (p_0^m)(1-p_0^m)(5^2)$$

$$20.25 = 20 - 20p_0^m + 25p_0^m - 25(p_0^m)^2$$

$$25(p_0^m)^2 - 5p_0^m + .25 = 0$$

$$p_0^m = \frac{5 \pm \sqrt{25-(4)(25)(.25)}}{50} = \frac{5}{50} = \frac{1}{10}$$

⑥ $E(X) = \frac{b-a}{2} = \frac{1000}{2} = 500$ $\quad LER = \frac{E(X \wedge 200)}{E(X)}$

$E(X \wedge 200) = \frac{200}{1000}\left(\frac{200-0}{2}\right) + \frac{800}{1000}(200) = 20 + 160 = 180$

Alternatively

$E(X \wedge 200) = \int_0^{200} x f(x)\, dx + 200(1 - F(200))$

$= \int_0^{200} x \frac{1}{1000}\, dx + 200(.80) = \left.\frac{x^2}{2000}\right|_0^{200} + 160$

$= 20 + 160 = 180$

⑦ If $n = 5$ or less, S cannot exceed 20 $\quad \uparrow (5)(4) = 20$

If $n = 6$ and all claims are an amount of 4

total claims $= (4)(6) = 24$

$Pr(24) = \left(\frac{6}{21}\right)(.5)^6$

If $n = 6$ and there are 5 claims of 4 and on claim of 1 then total claims $= 21$ $\quad Pr(21) = \left(\frac{6}{21}\right)(.5)^5(.5)\binom{6}{1}$

Otherwise claims are less than 20

$E(\text{Stop Loss at } 20.5) = (24 - 20.50)Pr(24) + (21 - 20.50)(Pr(21))$

$= (3.5)\left(\frac{6}{21}\right)(.5)^6 + (.5)\left(\frac{6}{21}\right)(.5)^5(.5)(6)$

$= 0.0290$

(8) $E(X) = E(E(X|\theta)) = E(\alpha\theta) = 2E(\theta) = 2\lambda = 2.4$

so $\lambda = 1.2$

$Var(X) = E[Var(X|\theta)] + Var[E(X|\theta)]$

$= E(\alpha\theta^2) + Var(\alpha\theta)$

$= 2E(\theta^2) + 4\,Var(\theta)$

$= 2[1.2 + 1.2^2] + 4(1.2)$

$= 10.08$

$Var(\theta) = E(\theta^2) - [E(\theta)]^2$

$\lambda = E(\theta^2) - \lambda^2$

$E(\theta^2) = \lambda + \lambda^2$

(9) $\text{Kurtosis} = \dfrac{\mu_4}{\sigma^4}$ $\quad E(X) = \dfrac{\Sigma x_i}{10} = \dfrac{50}{10} = 5$

$$Var(X) = \sigma^2 = \frac{2(1-5)^2 + 3(2-5)^2 + 2(0)^2 + 2(6-5)^2 + (20-5)^2}{10}$$

$= 28.6$

$$\mu_4 = \frac{2(1-5)^4 + 3(2-5)^4 + 2(0^4) + 2(6-5)^4 + (20-5)^4}{10}$$

$= 5138.2$

$$\text{Kurtosis} = \frac{5138.2}{(28.6)^2} = 6.2817$$

(10) $E(X) = \theta = 1000$

$$(1+r)\left(E\left(X \wedge \frac{u}{1+r}\right)\right) = 1000$$

$$(1.25)(\theta)\left(1 - e^{-\frac{u}{1.25\theta}}\right) = (1.25)(1000)\left(1 - e^{-\frac{u}{1250}}\right) = 1000$$

$$\therefore \quad 1.25 - 1.25\,e^{-\frac{u}{1250}} = 1$$

$$\frac{.25}{1.25} = e^{-\frac{u}{1250}}$$

$$\ln(.2) = -\frac{u}{1250}$$

$$u = \ln(.2)(-1250) = 2011.80$$