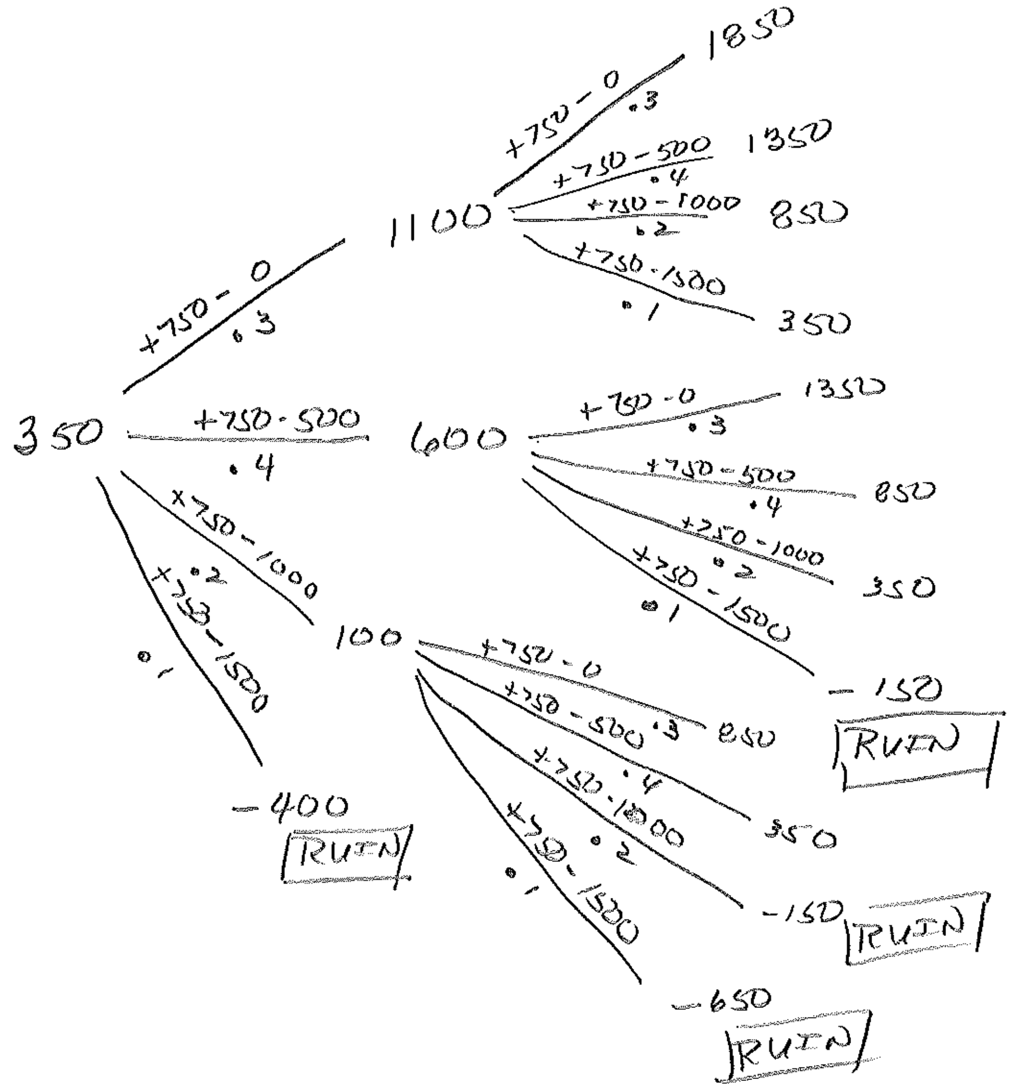
# TEST 2 - Fall 2008

①




RUIN = .2

$+(.4)(.1)+$

$(.2)(.2+.1)$

$= 0.20$

SURVIVAL

= 1 - RUIN

= 1 - .2

$= \underline{0.80}$

②

$$F_{20}(x) = \frac{c_j - x}{c_j - c_{j-1}} F(c_{j-1}) + \frac{x - c_{j-1}}{c_j - c_{j-1}} F(c_j)$$

$$= \frac{100 - x}{100 - 50}\left(\frac{14}{20}\right) + \frac{x - 50}{100 - 50}\left(\frac{18}{20}\right)$$

$$= \frac{1400 - 14x + 18x - 900}{1000} = \underline{\underline{.5 + .004x}}$$

$$f_{20}(x) = \frac{d}{dx} F_{20}(x) = \underline{\underline{.004}} \quad \text{or} \quad \frac{n_j}{n(c_j - c_{j-1})} = \frac{4}{(20)(50)}$$

$$= \underline{\underline{0.004}}$$

(3) $$\text{Mean} = \frac{8\left(\frac{20-0}{2}\right) + 6\left(\frac{50-20}{2}\right) + 4\left(\frac{100-50}{2}\right) + 2\left(\frac{200-100}{2}\right)}{20}$$

$$= \underline{\underline{44.5}}$$

$$\text{Var} = E(x^2) - \{E(x)\}^2$$

$$E(x^2) = \int_0^{20} x^2 f(x)\,dx + \int_{20}^{50} x^2 f(x) + \int_{50}^{100} x^2 f(x) + \int_{100}^{200} x^2 f(x)$$

but $f(x) = \frac{n_j}{n(c_j - c_{j-1})}$

$$= \int_0^{20} x^2 \frac{8}{(20)(20)}\,dx + \int_{20}^{50} x^2 \frac{6}{(20)(30)}\,dx + \int_{50}^{100} x^2 \frac{4}{(20)(50)}\,dx + \int_{100}^{200} x^2 \frac{2}{(20)(100)}\,dx$$

$$= .02 \left.\frac{x^3}{3}\right|_0^{20} + .01 \left.\frac{x^3}{3}\right|_{20}^{50} + .004 \left.\frac{x^3}{3}\right|_{50}^{100} + .001 \left.\frac{x^3}{3}\right|_{100}^{200}$$

$$= 53.\overline{3} + 390 + 1166.\overline{66} + 2333.\overline{33} = 3943.\overline{3}$$

$$\text{Var} = 3943.\overline{3} - (44.5)^2 = 1963.08\overline{3}$$

(4) $\hat{H}(1) = \frac{40}{100} = .4$ ; $\hat{H}(2) = \frac{40}{100} + \frac{30}{60} = .90$

$$\hat{H}(3) = \frac{40}{100} + \frac{30}{60} + \frac{A}{30} = .90 + \frac{A}{30}$$

$$\hat{S}(3) = e^{-\hat{H}(3)} \Rightarrow 0.2725 = e^{-0.90 - \frac{A}{30}}$$

$$\therefore \ln(0.2725) = -0.90 - \frac{A}{30} = 1.300 = 0.90 + \frac{A}{30}$$

$$\Rightarrow A = \underline{\underline{12}}$$

(5) $E(\text{No. of Losses}) = r\beta = 8$

$E(N^P) = 4 \qquad v = 0.50$

$N^P \sim$ negative binomial with $r = 2$ & $\beta = (.5)(4) = 2$

$Var(N^P) = r(\beta)(1+\beta) = (2)(2)(3) = \underline{12}$

(6) Class 1 $\quad E(X) = \lambda = 3 \qquad Var(X) = \lambda = 3$

Class 2 $\quad E(X) = \alpha\theta = (3)(2) = 6$

$Var(X) = \alpha\theta^2 = (3)(2)(2) = 12$

$$Var[S] = 60[q\sigma^2 + (q)(1-q)\mu^2] + 40[q\sigma^2 + (q)(1-q)\mu^2]$$

$$= 60[(.1)(3) + (.1)(.9)(3^2)] + 40[(.2)(12) + (.2)(.8)(6^2)]$$

$$= 393$$

(7) $\bar{X} = \frac{\Sigma X}{n} = \frac{223}{10} = 22.3$

$$S_x^2 = \frac{\Sigma X^2 - n\bar{X}^2}{n-1} = \frac{5693 - 10(22.3)^2}{9} = 80.011$$

$$Z = \left| \frac{\bar{X} - \mu}{\sqrt{\frac{S_x^2}{n}}} \right| = \frac{22.3 - 17.5}{\sqrt{\frac{80.011}{10}}} = 1.6969$$

critical value from table 1.96 since it is a two tailed test

We cannot reject Ho since $z \leq 1.96$

$p = (1 - .9554)2 = .0892$

(8) $MSE = Var(\hat{\theta}) + [bias_{\hat{\theta}}(\theta)]^2$

$$bias_{\hat{\theta}}(\theta) = E(\hat{\theta}|\theta) - \theta = E\left[\frac{2\Sigma X}{2n-1}\right] - \theta$$

$$= \frac{2}{2n-1} E[X_1 + X_2 + \dots + X_{10}] - 200 = \frac{200}{2n-1} \cdot 10\,E(X) - 200$$

$$= \frac{200}{2(10)-1} \cdot 10\,(200) - 200 = 10.52631579$$

$$Var\left[\frac{2\Sigma X}{2n-1}\right] = \left(\frac{2}{2n-1}\right)^2 Var(X_1 + X_2 + \dots + X_{10}) =$$

$$\left(\frac{2}{2(10)-1}\right)^2 (10) Var(X) = \frac{4\,(10)}{361}\theta^2 = \frac{40(200)^2}{361} = 4432.132964$$

$$MSE = 4432.13 + (10.526)^2 = \underline{\underline{4542.94}}$$

(9) Unbiased estimate of mean is sample mean

$$\bar{X} = \frac{10+15+20+30+40}{5} = 23$$

unbiased estimate of sample variance

$$= \frac{\Sigma (x_i - \bar{x})^2}{n-1} =$$

$$\frac{(10-23)^2 + (15-23)^2 + (20-23)^2 + (30-23)^2 + (40-23)^2}{4}$$

$$= \underline{145}$$

(10)

$$f(2000) = F(2250) - F(1750)$$

$$= \left[1 - e^{-\left(\frac{2250}{1000}\right)^2}\right] - \left[1 - e^{-\left(\frac{1750}{1000}\right)^2}\right]$$

$$= e^{-\left(\frac{1750}{1000}\right)^2} - e^{-\left(\frac{2250}{1000}\right)}$$

$$= .040440907$$