Stat 479
Fall 2009
Quiz 1
September 3, 2009

1. You are given that $F_x(x) = 1 - (100/x)^4$ for $x \geq 100$.

   You are also given that $f_y(y)$ is:

y	$f_y(y)$
100	0.4
200	0.3
300	0.2
400	0.1

   Calculate Var(Y) – Var(X).

$$f_x(x) = \frac{d}{dx} F_x(x) = 4(100)^4 x^{-5}$$

$$E(X) = \int_{100}^{\infty} 400(100)^3 x^{-5} x \, dx$$

$$= \int_{100}^{\infty} 400(100)^3 x^{-4} dx$$

$$= 400(100)^3 \frac{x^{-3}}{-3} \Big|_{100}^{\infty} = \frac{400}{3}$$

$$E(X^2) = \int_{100}^{\infty} 400(100)^3 x^{-5} x^2 dx = \int_{100}^{\infty} 400(100)^3 x^{-3} dx$$

$$= 400(100)^3 \frac{x^{-2}}{-2} \Big|_{100}^{\infty} = \frac{(400)(100)}{2} = 20{,}000$$

$$Var(X) = E(X^2) - (E[X])^2 = 20000 - \left(\frac{400}{3}\right)^2 = 2222.2\overline{2}$$

$$E(Y) = 100(.4) + 200(.3) + 300(.2) + 400(.1) = 200$$

$$E(Y^2) = (100)^2(.4) + (200)^2(.3) + (300)^2(.2) + (400)^2(.1) = 50000$$

$$Var(Y) = E(Y^2) - [E(Y)]^2 = 50000 - 40000 = 10{,}000$$

$$Var(Y) - Var(X) = 10{,}000 - 2222.2\overline{2} = 7777.7\overline{7}$$

2. Automobile losses are distributed as a Gamma distribution with parameters α and θ. The expected loss is 800 and the variance of the loss is 160,000.

Determine the mode.

$E(X) = \alpha\theta$

$Var(X) = \alpha\theta^2$

$$\theta = \frac{Var(X)}{E(X)} = \frac{160{,}000}{800} = 200$$

$$\alpha = \frac{E(X)}{\theta} = \frac{800}{200} = 4$$

$$\text{Mode} = \theta(\alpha - 1) = 200(3) = \underline{\underline{600}}$$