

**Stat 479**  
**Fall 2009**  
**Quiz 10**  
**December 3, 2009**

1. You have the following sample from a distribution:
- $\underbrace{6, 6, 8, 10, 10, 10, 14, 14, 16, 19}_{\text{6th value}}$        $\underbrace{7\text{th value}}_{\text{7th value}}$

$\theta_M$  is the estimated parameter for an exponential distribution using the Method of Moments.

$\theta_P$  is the estimated parameter for an exponential distribution using the Method of Percentile Matching using the 60<sup>th</sup> percentile.

Calculate  $1000(\theta_P - \theta_M)$ .

$$\hat{\theta}_M = \bar{X} = \frac{6+6+8+10+10+10+14+14+16+19}{10} = 11.3$$

For Percentile matching we must find  
the 60<sup>th</sup> percentile of the smoothed  
empirical distribution

$$n = 10 \quad j = \lfloor (n+1)(p) \rfloor = \lfloor (11)(.6) \rfloor = \lfloor 6.6 \rfloor = 6$$

$$h = .6 \quad \begin{matrix} \downarrow & \text{jth value} \\ \pi_p = (1 - .6)(\underset{h}{10}) + (.6)(\underset{h}{14}) = 12.4 & \downarrow \end{matrix} \quad \begin{matrix} \downarrow & \text{j+1 value} \end{matrix}$$

$$F(12.4) = .6 = 1 - e^{-\frac{12.4}{\hat{\theta}_P}} \Rightarrow \hat{\theta}_P = 13.5328$$

$$1000 (\theta_P - \theta_M) = 1000 (13.5328 - 11.3)$$

$$= 2232.8$$

2. One hundred laptop computers are observed for a period of 12 months. Thirty laptops malfunction during the observation period, with the following distribution:

Time Till Malfunction in Months	Number of Malfunctions
1	8
2	6
3	0
4	0
5	1
6	0
7	1
8	2
9	2
10	3
11	3
12	4

The remaining seventy laptops are still functioning at the end of 12 months.

The lifetime of a laptop is believed to follow an exponential survival function with mean of  $\theta$ .

Calculate the maximum likelihood estimate of  $\theta$ .

$$\hat{\theta} = \frac{\text{Total DAYS}}{\text{Number of Uncensored PAYMENTS}}$$

$$= \frac{(8)(1) + (6)(2) + (5)(1) + (7)(1) + 8(2) + 9(2) + (10)(3) + (11)(3) + (12)(4) + (12)(70)}{30}$$

$$= \frac{1017}{30} = 33.9$$

Alternatively, you could do it from first principles.

These are the 70 laptops still working