1. Losses from a policy covering emergency room visits are distributed as a Pareto distribution with $\alpha = 3$ and $\theta = 1000$.

    The insurance company wants to impose a deductible such that the expected cost per emergency room visit under the policy is reduced to 50%. In other words:

    $$E[(X-d)_+] = 0.5E[X]$$

    Determine d.

$$E(X) = \frac{\theta}{\alpha - 1} = \frac{1000}{3-1} = 500$$

$$E[(X \wedge d)] + E[(x-d)_+] = E[X]$$

$$E[(x \wedge d)] + \frac{1}{2} E[X] = E[X]$$

$$E[(x \wedge d)] = \frac{1}{2} E[X] = \frac{1}{2}(500) = 250$$

$$E[(x \wedge d)] = \frac{\theta}{\alpha - 1}\left(1 - \left(\frac{\theta}{\theta + d}\right)^{\alpha - 1}\right)$$

$$250 = \frac{1000}{3-1}\left(1 - \left(\frac{1000}{1000+d}\right)^{3-1}\right)$$

$$250 = 500\left(1 - \left(\frac{1000}{1000+d}\right)^2\right)$$

$$\frac{1}{2} = 1 - \left(\frac{1000}{1000+d}\right)^2$$

$$\sqrt{\frac{1}{2}} = \frac{1000}{1000+d} \Rightarrow d = \frac{1000}{\sqrt{\frac{1}{2}}} - 1000 = 414.21$$

2. The random variable X is uniformly distributed between 20 and z.

TVaR$_{.80}$(X) = 155.

Determine k so that the standard deviation principle is also equal to 155.

$$TVaR_p(x) = \frac{b + a + p(b-a)}{2}$$

$p = .80$
$a = 20$
$b = z$

$$= \frac{z + 20 + (.8)(z-20)}{2} = 155$$

$$\Rightarrow 1.8z + 4 = 310 \Rightarrow z = 170$$

$$E(X) = \frac{b+a}{2} = \frac{170 + 20}{2} = 95$$

$$Var(X) = \frac{(b-a)^2}{12} = \frac{(150)^2}{12} = 1875$$

$$\sigma = \sqrt{1875} = 43.30127019$$

Standard Deviation PRINCIPLE

$$= \mu + k\sigma = 155$$

$$95 + k(43.30127019) = 155$$

$$k = \frac{155 - 95}{43.30127019} = 1.38564$$