Stat 479
Fall 2009
Quiz 4
October 1, 2009

1. Under a zero-modified geometric distribution, Var(N) = 88.

   If $p_0^M = 1/5$, calculate β.

Variance for zero modified

$$= (1-p_0^m)[\text{Variance of zero truncated}] + (p_0^m)(1-p_0^m)[\text{mean of zero truncated}]^2$$

$$= (1-1/5)(\beta)(1+\beta) + (1/5)(1-1/5)(1+\beta)^2 = 88$$

$$= 4/5(\beta+\beta^2) + 4/25(1+2\beta+\beta^2) = 88$$

$$= 20\beta + 20\beta^2 + 4 + 8\beta + 4\beta^2 - (88)(25) = 0$$

$$= 24\beta^2 + 28\beta - 2196 = 0$$

$$\beta = \frac{-28 \pm \sqrt{(28)^2 - (4)(24)(-2196)}}{48}$$

$$= 9$$

2. Losses are distributed as a Pareto with $\alpha = 2$ and θ.

If losses are subject to an ordinary deductible of 20,000, the expected value **per payment** is 30,000.

If losses are subject to a franchise deductible of 20,000, calculate the expected value **per loss**.

$$E(X) \text{ per payment with ordinary deductible} = \frac{E(X) - E(X \wedge d)}{1 - F(d)}$$

$$= \frac{\frac{\theta}{\alpha-1} - \frac{\theta}{\alpha-1}\left[1 - \left(\frac{\theta}{\theta+20000}\right)^{\alpha-1}\right]}{1 - \left[1 - \left(\frac{\theta}{\theta+20000}\right)^{\alpha}\right]}$$

$$= \frac{\frac{\theta}{1}\left(\frac{\theta}{\theta+20000}\right)}{\left(\frac{\theta}{\theta+20000}\right)^{2}} = \frac{\theta(\theta+20000)}{\theta}$$

$$= \theta + 20{,}000 = 30000$$

$$\therefore \theta = 10{,}000$$

$$E(X) \text{ per loss with franchise deductible} = E(X) - E(X \wedge d) + d\left[1 - F(d)\right]$$

$$= \frac{\theta}{\alpha-1} - \frac{\theta}{\alpha-1}\left[1 - \left(\frac{\theta}{\theta+20000}\right)^{\alpha-1}\right] + 20000\left[\frac{\theta}{\theta+20000}\right]^{2}$$

$$= 10000\left[\frac{10000}{30000}\right]^{1} + 20000\left[\frac{10000}{30000}\right]^{2}$$

$$= \frac{10000}{3} + \frac{20000}{9} = 5555.5\overline{5}$$