Stat 479
Fall 2009
Quiz 7
October 29, 2009

1. An urn contains four balls. Each ball has a unique number on it. The numbers on the balls are 1, 2, 3, and 4.

   One ball is drawn from the urn and is used to estimate the mean of the numbers on the ball.

   Calculate the Mean Square Error of this estimator.

$$MSE = E(\hat{\theta} - \theta)^2$$

$$= E(\hat{\theta}^2 - 2\theta\hat{\theta} + \theta^2)$$

θ = Mean of number on ball = $\frac{1+2+3+4}{4} = 2.5$

$$E(\hat{\theta}) = \tfrac{1}{4}(1) + \tfrac{1}{4}(2) + \tfrac{1}{4}(3) + \tfrac{1}{4}(4) = 2.5$$

$$E(\hat{\theta}^2) = \tfrac{1}{4}(1^2) + \tfrac{1}{4}(2^2) + \tfrac{1}{4}(3^2) + \tfrac{1}{4}(4^2) = 7.5$$

$$MSE = E(\hat{\theta}^2) - 2(2.5)E(\hat{\theta}) + (2.5)^2 =$$

$$= 7.5 - 2(2.5)(2.5) + (2.5)^2 = 1.25$$

<u>OR</u> $MSE = Var(\hat{\theta}) + [bias_{\hat{\theta}}(\theta)]^2$

but the estimator is $\bar{X}$ so it is unbiased

so $MSE = Var(\hat{\theta})$

$$= E(\hat{\theta}^2) - [E(\hat{\theta})]^2$$

$$= 7.5 - (2.5)^2 = 1.25$$

From above.

2. You are given:

$\theta = 6$

$E[\hat{\theta}^2] = 17$

$MSE_{\hat{\theta}}(\theta) = 5$

Calculate $E[\hat{\theta}]$.

$$MSE = Var(\hat{\theta}) + [bias_{\hat{\theta}}(\theta)]^2$$

$$= E(\hat{\theta}^2) - [E(\hat{\theta})]^2 + [E(\hat{\theta}) - \theta]^2$$

$$= E(\hat{\theta}^2) - [E(\hat{\theta})]^2 + [E(\hat{\theta})]^2 - 2\theta E(\hat{\theta}) + \theta^2$$

$$= E(\hat{\theta}^2) - 2\theta E(\hat{\theta}) + \theta^2 = 5$$

$$= 17 - (2)(6)E(\hat{\theta}) + 6^2 = 5$$

$$= -12E(\hat{\theta}) = -48$$

$$\Rightarrow E(\hat{\theta}) = 4$$