1. Amstutz Ant Farm is studying the life expectancy of ants. The farm's actuary isolates 100 ants and records the following data:

Number of Days till Death	Number of Ants Dying	r_j
1	6	100
2	10	94
3	12	84
4	20	72
5	35	
6	10	
7	3	
8	2	
9	1	
10	1	

Estimate $\hat{S}(4)$ using the Nelson-Åalen estimator.

$$\hat{H}(4) = \frac{6}{100} + \frac{10}{94} + \frac{12}{84} + \frac{20}{72} = 0.587017899$$

$$\hat{S}(4) = e^{-\hat{H}(4)} = e^{-0.587017899} = 0.556$$

2. You are given the following frequency distribution:

Number of Claims	Probability
0	0.20
1	0.25
2	0.40
3	0.15

You are also given the following severity distribution:

Amount of Claim	Probability
100	0.50
200	0.40
500	0.10

Calculate $F_S(350)$.

$$F_S(350) = f_S(0) + f_S(100) + f_S(200) + f_S(300)$$

$$f_S(0) = Pr(N=0) = 0.2$$

$$f_S(100) = Pr(N=1) \cdot Pr(X=100) = (.25)(.5) = 0.125$$

$$f_S(200) = Pr(N=1) \cdot Pr(X=200) + Pr(N=2) \cdot Pr(\text{Both } X=100)$$
$$= (.25)(.40) + (.4)(.5)^2 = 0.20$$

$$f_S(300) = Pr(N=2) \cdot Pr(X=100 \text{ and } X=200) + Pr(N=3) \cdot Pr(\text{All } X=100)$$
$$= (.4)(.5)(.4)(2) + (.15)(.5)^3 = .17875$$

$$F_S(350) = 0.20 + 0.125 + 0.20 + 0.17875$$
$$= .70375$$

3. You are given the following frequency distribution:

Number of Claims	Probability
0	0.20
1	0.25
2	0.40
3	0.15

You are also given the following severity distribution:

Amount of Claim	Probability
100	0.50
200	0.40
500	0.10

Calculate the Net Stop Loss premium for an aggregate deductible of 1000.

We can only exceed the deductible if we have
3 claim for 500 each
or 3 claims with 2 for 500 and one of either 100 or 200

Net Stop Loss Premium=

$$(1500-1000)\,Pr(N=3)\cdot Pr(\text{All } X=500)$$
$$+(1200-1000)\,Pr(N=3)\cdot Pr(\text{Two } X=500 \text{ and } X=200)$$
$$+(1100-1000)\,Pr(N=3)\cdot Pr(\text{Two } X=500 \text{ and } X=100)$$
$$=500(.15)(.1)^3+(200)(.15)(.1)^2(.4)(3)$$
$$+(100)(.15)(.1)^2(.5)(3)=\underline{0.66}$$

4. The number of claims under a five year medical insurance policy is distributed as a Poisson distribution with a mean of 100. The severity of claims is distributed as a Pareto distribution with $\theta = 1000$ and $\alpha = 3$.

Calculate the 90% confidence interval for the expected aggregate claims if aggregate claims are assumed to follow a normal distribution.

$E(N) = 100 \qquad Var(N) = 100$

$E(X) = \frac{\theta}{\alpha - 1} = \frac{1000}{3-1} = 500$

$Var(X) = \frac{\theta^2 \alpha}{(\alpha-1)^2(\alpha-2)} = \frac{(1000)^2 (3)}{(2)^2} = 750{,}000$

$E(S) = E(N)\,E(X) = (100)(500) = 50{,}000$

$Var(S) = E(N)\,Var(X) + Var(N)[E(X)]^2$

$= (100)(750{,}000) + (100)(500)^2$

$= 100{,}000{,}000$

90% confidence interval =

$E(S) \pm \Phi(.95)\sqrt{Var(S)} =$

$50{,}000 \pm 1.645\sqrt{100{,}000{,}000}$

$= 50000 \pm 1.645(10{,}000)$

$(33{,}550,\ 66{,}450)$

5. Sahlin Health Insurance Company is providing all students in STAT 479 with a health insurance policy. The health insurance policy provides payment if a person in this class is hospitalized with H1N1.

The following table is a census of the students covered and their probability of claim:

Gender	**Number of Students**	**Probability of Claim**
Female	12	0.05
Male	16	0.10

If a person is hospitalized, claims are distributed as a Gamma distribution with $\alpha = 2$ and $\theta = 2000$ without regard to gender.

Claims are independent.

Calculate E(S) and Var(S).

$$E(S) = \sum q_i \mu_i$$

$$= (12)(.05)(2)(2000) + (16)(.1)(2)(2000)$$

$$= \underline{8800}$$

$$Var(S) = \sum q_i \sigma_i^2 + q_i(1-q_i)\mu_i^2$$

$$= (12)[(.05)(2)(2000)^2 + (.05)(.95)(4000)^2]$$

$$+ 16[(.1)(2)(2000)^2 + (.1)(.9)(4000)^2]$$

$$= 49{,}760{,}000$$

6. Disability claims are assumed to be distributed uniformly from zero to θ. A sample of four disability claims are drawn and used to estimate θ by taking the maximum of the sample as the estimate of θ.

$n=4$

The true distribution of disability claims is a uniform distribution from zero to 1000.

Calculate the Mean Square Error of our estimator.

$\theta = 1000$

$$MSE = Var(\hat{\theta}) + (bias_{\hat{\theta}}(\theta))^2$$

$$Var(\hat{\theta}) = \frac{n\theta^2}{(n+2)(n+1)^2} = \frac{4(1000)^2}{6\cdot(5)^2} = 26{,}666.67$$

$$bias = (E(\hat{\theta}) - \theta)^2 = \left(\frac{n\theta}{n+1} - \theta\right)^2$$

$$= \left(\frac{4(1000)}{5} - 1000\right)^2 = (-200)^2 = 40{,}000$$

$$MSE = 26{,}666.67 + 40{,}000 = 66{,}666.67$$

7. Dental claims are distributed exponentially with $\theta = 100$. You want to discretize the claims using a span of 20 using both the Method of Rounding and the Method of Moment Matching where you will match the mean.

Calculate the probability assigned to the range that includes 100 under each method.

Ranges with $h=20$

0 - 10
10 - 30
30 - 50
50 - 70
70 - 90
90 - 110

Method of Rounding

$$f_5 = F(110) - F(90)$$
$$= \left(1-e^{-\frac{110}{100}}\right) - \left(1-e^{-\frac{90}{100}}\right)$$
$$= e^{-.9} - e^{-1.1} = 0.073698576$$

Method of Moment matching

$$f_5 = \frac{2E(X \wedge 100) - E(X \wedge 80) - E(X \wedge 120)}{20}$$
$$= \frac{2(100)\left(1-e^{-\frac{100}{100}}\right) - (100)\left(1-e^{-\frac{80}{100}}\right) - 100\left(1-e^{-\frac{120}{100}}\right)}{20}$$
$$= 0.073821468$$

8. You are given the following sample data:

X: 5, 6, 8, 10, 10, 10, 13, 17, 20, 21

You are also given:

N = 10
Sum of X_i = 120
Sum of X_i^2 = 1724

You complete Hypothesis Testing with:

H_0: The mean is 10.

H_1: The mean is not 10.

Calculate the z statistic, the critical value(s) assuming a significance level of 30%, and the p value. State your conclusion with regard to the Hypothesis Testing.

$$\bar{X} = \frac{120}{10} = 12$$

$$\widehat{Var}(x) = \frac{\Sigma(x_i^2) - N(\bar{x}^2)}{N-1} = \frac{1724 - 10(12)^2}{9} = 31.55\bar{5}$$

$$z = \left| \frac{12-10}{\sqrt{\frac{31.55\bar{5}}{10}}} \right| = 1.1258799$$

critical values = $\pm$ 1.036

We reject Ho because 1.1258799 > 1.036

p values = $2(1 - \Phi(1.1258799)) = .26$

9. Troup Trucking Company suffered 100 accidents last year. The following table summarizes the amount of losses on those 100 accidents:

Amount of Loss	Number of Accidents
0 to 10,000	15
10,000 to 25,000	25
25,000 to 100,000	35
100,000 to 250,000	20
Over 250,000	5

Calculate $F_{100}(x)$ for losses between 25,000 and 100,000 using the Ogive.

$F(25{,}000) = \frac{40}{100} = .4$

$F(100{,}000) = \frac{75}{100} = .75$

Ogive is linear interpolation between these values

$$\frac{100000 - x}{75{,}000}(.40) + \frac{x - 25{,}000}{75{,}000}(.75)$$

$$= \frac{40{,}000 - .4x + .75x - 18750}{75000}$$

$$= \frac{21250 + .35x}{75{,}000}$$

10. Troup Trucking's main competitor, Huang Hauling LTD, has the following accident information from last year:

Amount of Loss	Number of Accidents	Total Loss
0 to 10,000	5	30,000
10,000 to 25,000	8	130,000
25,000 to 100,000	12	840,000
100,000 to 250,000	3	460,000
Over 250,000	2	740,000

Huang Hauling buys truck insurance which will pay claims up to a limit of 100,000 per claim.

Using the distribution of claims from last year, calculate the expected value per payment for the insurance company.

TOTAL PAYMENTS =

$$30,000 + 130,000 + 840,000 + 3(100,000) + 2(100,000)$$

$$= 1,500,000$$

Number of Payments = 30

$$\text{Expected Value} = \frac{1,500,000}{30} = 50,000$$