# Quiz 2

## STAT 479

**September 9, 2010**

1. Dental claims during a year are distributed as a 2 point mixture distribution with a weight of 70% for Distributon 1 and 30% for Distribution 2.

   Distribution 1 is a Pareto distribution with $\alpha = 3$ and $\theta = 400$

   Distribution 2 is a Gamma distribution with $\alpha = 2$ and $\theta = 2000$.

   An insurance company has 2000 independent dental policies which pay 100% of all claims.

   Using the normal approximation, estimate the probability that total claims will exceed 2.7 million.

FOR PARETO — weight 0.70

$$E(X) = \frac{\theta}{\alpha-1} = \frac{400}{2} = 200$$

$$E(X^2) = \frac{2\theta^2}{(\alpha-1)(\alpha-2)} = \frac{2(400)^2}{(2)(1)} = (400)^2$$

FOR GAMMA — weight 0.30

$$E(X) = \alpha\theta = (2)(2000) = 4000$$

$$E(X^2) = \theta^2(\alpha+1)(\alpha) = (2000)^2(3)(2) = (6)(2000)^2$$

$$E[Y] = (.7)(\text{PARETO}) + (.3)(\text{GAMMA}) = (.7)(200) + (.3)(4000) = 1340$$

$$E[Y^2] = (.7)(\text{PARETO}) + (.3)(\text{GAMMA}) = (.7)(400^2) + (.3)(6)(2000^2) = 7,312,000$$

$$Var(Y) = E(Y^2) - (E[Y])^2 = 7,312,000 - (1340)^2 = 5,516,400$$

$$E(S) = 2000(1340) \qquad Var(S) = (2000)(5,516,400)$$

$$Pr(S > 2,700,000) = Pr\left(Z > \frac{2,700,000 - 2000(1340)}{\sqrt{(2000)(5,516,400)}}\right)$$

$$= Pr(Z > 0.1904) = 1 - 0.5753 = \underline{\underline{0.4247}}$$

2. Cancer claims follow an exponential distribution with parameter θ.

$VaR_{0.75}(X) = 170.5142$

Calculate k so that the standard deviation principle is equal to $TVaR_{0.75}(X)$.

From Tables

$E(x) = \theta^1(1!) = \theta$

$E(x^2) = \theta^2(2!) = 2\theta^2$

$Var(x) = E(x^2) - [E(x)]^2 = 2\theta^2 - \theta^2 = \theta^2$

$\sigma = \sqrt{Var\, x} = \theta$

$(TVaR)_{.75}(x) = -\theta(\ln(1-p)) + \theta = -\theta \ln(.25) + \theta$

so

$\mu + k\sigma = TVaR_{.75}(x)$

$\theta + k\theta = -\theta \ln(.25) + \theta$

so $k = -\ln(.25)$

$= 1.38629$

You can also solve by finding $\theta = 123$ and then doing the rest of the work.