

Quiz 4
STAT 479
 September 28, 2010

1. Losses prior to any deductible for a health insurance policy are distributed as a Pareto distribution with $\theta = 1000$ and $\alpha = 5$. Losses are subject to a franchise deductible of d .

The expected value per payment after the deductible is 820.

Calculate d .

$$\begin{aligned}
 E(Y^P) &= \frac{E(X) - E(X \wedge d) + d(1 - F(d))}{1 - F(d)} \\
 &= \frac{\frac{\theta}{\alpha - 1} - \frac{\theta}{\alpha - 1} \left[1 - \left(\frac{\theta}{d + \theta} \right)^{\alpha - 1} \right]}{1 - \left[1 - \left(\frac{\theta}{d + \theta} \right)^{\alpha} \right]} + d \\
 &= \frac{\frac{\theta}{\alpha - 1} \left[\left(\frac{\theta}{d + \theta} \right)^{\alpha - 1} \right]}{\left(\frac{\theta}{d + \theta} \right)^{\alpha}} + d \\
 &= \frac{\theta}{\alpha - 1} \left[\frac{d + \theta}{\theta} \right] + d = \frac{d + \theta}{\alpha - 1} + d \\
 820 &= \frac{d + 1000}{4} + d = \frac{5}{4}d + 250 \\
 d &= (820 - 250) \cdot 0.8 = \underline{\underline{456}}
 \end{aligned}$$

2. Losses follow an exponential distribution with a mean of 1000. The insurance company wants to implement an ordinary deductible that will result in a loss elimination ratio of 0.5.

Calculate the ordinary deductible to be implemented.

$$\Theta = 1000 \quad \text{LER} = 0.5$$

$$\text{LER} = \frac{E(X \wedge d)}{E(X)} = 0.5$$

$$\Rightarrow E(X \wedge d) = (0.5)(E(X)) = (0.5)(1000) = 500$$

From Tables

$$E(X \wedge d) = \Theta \left(1 - e^{-\frac{d}{\Theta}}\right)$$

$$1000 \left(1 - e^{-\frac{d}{1000}}\right) = 500$$

$$1 - e^{-\frac{d}{1000}} = 0.5$$

$$e^{-\frac{d}{1000}} = 0.5$$

$$\frac{d}{1000} = -\ln(0.5)$$

$$d = (-\ln(0.5))(1000)$$

$$= 693.15$$