Stat 479

Test 1

September 30, 2010

1. The number of student from Interest Theory that visit my office during office hours is distributed as a Poisson distribution with a mean of 2 per hour.

   The number of students from Loss Models that visit my office during office hours is distributed as a Poisson distribution with a mean of 1 per hour.

   The number of students from Life Contingencies that visit my office during office hours is distributed as a Poisson distribution with a mean of 0.5 per hour.

   Calculate the probability that more than 3 students (from any class) visit my office during office hours from 2:00 pm to 4:00 pm.

$\lambda = 2 + 1 + 0.5 = 3.5$ per hour

$\lambda = 7$ for two hours

$P(\# > 3) = 1 - Pr(\# \leq 3) =$

$1 - P_0 - P_1 - P_2 - P_3$

$= 1 - e^{-7} - e^{-7}\left(\frac{7^1}{1!}\right) - e^{-7}\left(\frac{7^2}{2!}\right) - e^{-7}\left(\frac{7^3}{3!}\right)$

$= 0.91823$

2. Losses in 2009 are distributed as a Pareto distribution with $\alpha = 3$ and $\theta = 10{,}000$. Yan Health Insurance Company sell a policy that covers these losses with a franchise deductible of 2000 during 2009.

Losses in 2010 increase by 10%. During 2010, Yan will sell a policy covering the losses. However, instead of the franchise deductible used in 2009, the company will implement an ordinary deductible of d.

The expected value of per loss for Yan Health Insurance Company is the same in 2010 as it was in 2009.

Determine d.

2009

$$E(Y^L) = E[X] - E[X \wedge d] + d(1-F(d))$$

$$= \frac{\theta}{\alpha-1} - \frac{\theta}{\alpha-1}\left[1-\left(\frac{\theta}{\theta+d}\right)^{\alpha-1}\right] + d\left(\frac{\theta}{\theta+d}\right)^{\alpha}$$

$$= \frac{\theta}{\alpha-1}\left(\frac{\theta}{\theta+d}\right)^{\alpha-1} + d\left(\frac{\theta}{\theta+d}\right)^{\alpha} = \frac{10000}{2}\left(\frac{10000}{12000}\right)^{2} + 2000\left(\frac{10000}{12000}\right)^{3}$$

$$= 4629.63$$

2010 $\qquad \theta = 1.1\,(10000) = 11000$

$$E(Y^L) = E[X] - E[X \wedge d] = \frac{\theta}{\alpha-1} - \frac{\theta}{\alpha-1}\left[1-\left(\frac{\theta}{\theta+d}\right)^{\alpha-1}\right]$$

$$= \frac{\theta}{\alpha-1}\left[\frac{\theta}{\theta+d}\right]^{\alpha-1} = \frac{11000}{2}\left[\frac{11{,}000}{11{,}000+d}\right]^{2}$$

$$\therefore\ 5500\left(\frac{11000}{11000+d}\right)^{2} = 4629.63$$

$$\frac{11000}{11000+d} = \left(\frac{4629.63}{5500}\right)^{1/2}$$

$$d = \frac{11000}{\sqrt{\frac{4629.63}{5500}}} - 11000 = \underline{989.495}$$

3. The cost of an office visit to a doctor is distributed as a **single parameter** Pareto with $\alpha = 2$ and $\theta = 50$. The HMO for which the doctor works pays the doctor the cost of the office visit plus a bonus if the cost of the office visit is less than 80. The bonus is equal to 0.5(80-C) were C is the cost of the office visit.

Calculate the expected total payment (cost of visit plus bonus) to the doctor per office visit.

$$E(\text{COST}) = \frac{\alpha(\theta)}{\alpha-1} = \frac{2(50)}{1} = 100$$

$$E(\text{BONUS}) = 0.50\left[80 - E[X \wedge 80]\right]$$

$$= 0.5\left[80 - \left(\frac{2(50)}{1} - \frac{(1)(50)^2}{(2-1)(80)^{2-1}}\right)\right]$$

$$= 5.625$$

$$E[\text{TOTAL}] = 100 + 5.625 = \underline{\underline{105.625}}$$

4. The number of automobile accidents on Purdue campus during any given day is distributed as a zero modified Poisson distribution with λ = 3. The variance of the number of accidents is 3.98883.

Determine p_0^M given that $p_0^M \geq 0.35$.

$$\text{Var (zero mod)} = (1-p_0^m)\left(\frac{\lambda(1-(\lambda+1)e^{-\lambda})}{(1-e^{-\lambda})^2}\right) + (p_0^m)(1-p_0^m)\left[\left(\frac{\lambda}{1-e^{-\lambda}}\right)^2\right]$$

$$3.98883 = (1-p_0^m)\left[\frac{3(1-(4)e^{-3})}{(1-e^{-3})^2}\right] + (p_0^m)(1-p_0^m)\left(\frac{3}{1-e^{-3}}\right)^2$$

$$3.98883 = (1-p_0^m)(2.66091804) + (p_0^m)(1-p_0^m)(9.967830318)$$

$$3.98883 = 2.66091804 - 2.66091804\,p_0^m + 9.967830318\,p_0^m - 9.967830318(p_0^m)^2$$

$$\therefore\ 9.967830318(p_0^m)^2 - 7.30691227\,p_0^m + 1.32791196 = 0$$

$$p_0^m = \frac{7.30691227 \pm \sqrt{(-7.30691227)^2 - (4)(9.967830318)(1.32791196)}}{2(9.967830318)}$$

$$= 0.333 \text{ or } 0.400$$

since $p_0^m > 0.35 \Rightarrow \underline{\underline{0.400}}$

5. You are given $F(x) = \dfrac{x^2}{10,000}$ for $0 < x < 100$.

Calculate Var(X).

$$f(x) = F'(x) = \frac{2x}{10000} = \frac{x}{5000}$$

$$E(x) = \int_0^{100} x \, f(x)\, dx = \int_0^{100} \frac{x^2}{5000}\, dx$$

$$= \left.\frac{x^3}{15000}\right|_0^{100} = 66.\bar{6}$$

$$E(x^2) = \int_0^{100} x^2 \, f(x)\, dx = \int_0^{100} \frac{x^3}{5000}\, dx$$

$$= \left.\frac{x^4}{20,000}\right|_0^{100} = 5000$$

$$Var(x) = E(x^2) - [E(x)]^2 = 5000 - (66.\bar{6})^2$$

$$= \underline{\underline{555.\bar{5}}}$$

6. The random variable N is the number of failures per 100 iPhones in a given year. N is distributed as a Binomial distribution with m = 100 and q. Further, q is distributed uniformly between 0.2 and 0.5.

Calculate the Var(N).

$$Var(N) = E[Var\{N|Q\}] + Var[E(N|Q)]$$

$$= E[mq(1-q)] + Var[mq]$$

$$= E[100q(1-q)] + Var[100q]$$

$$= 100[E(q) - E(q^2)] + (100)^2 Var(q)$$

$$E(q) = \frac{a+b}{2} = \frac{0.2+0.5}{2} = 0.35$$

$$Var(q) = \frac{(b-a)^2}{12} = \frac{(.3)^2}{12} = 0.0075$$

$$E(q^2) = \frac{b^3-a^3}{(b-a)(3)} = 0.13$$

OR

$$E(q^2) = \int_{.2}^{.5} q^2\left(\frac{1}{.3}\right)dq = .13$$

or

$$Var(q) = E(q^2) - (E(q))^2 \Rightarrow E(q^2) = .0075 + (.35)^2 = 0.13$$

$$= 100(0.35 - 0.13) + (100)^2(.0075)$$

$$= \underline{\underline{97}}$$

7. Hennessy HMO insures 100 identical independent policyholders. Losses for each policyholder are distributed as a Gamma distribution with $\alpha = 4$ and θ.

Using the normal approximation, the probability that the total claims from all 100 policyholders will be less than 180,000 is 2.28%.

Calculate θ.

$E[X] = \alpha\theta = 4\theta$ $\qquad$ $E[S] = 100E[X] = 400\theta$

$Var[X] = \alpha\theta^2 = 4\theta^2$ $\qquad$ $Var[S] = 100\,Var[X] = 400\theta^2$

$$\Pr\left[Z < \frac{180{,}000 - 400\theta}{\sqrt{400\theta^2}}\right] = .0228$$

$\therefore\ Z = -2$

$$-2 = \frac{180{,}000 - 400\theta}{20\theta}$$

$-40\theta = 180000 - 400\theta$

$360\theta = 180{,}000$

$\theta = \underline{\underline{500}}$

8. You are given the following claims from last year:

10, 14, 35, 50, 50, 50, 72, and 103

These claims are used to form an empirical distribution.

Calculate μ, σ^2, and the coefficient of variation.

Empirical Distribution

10	$\frac{1}{8}$
14	$1/8$
35	$1/8$
50	$3/8$
72	$1/8$
103	$1/8$

$$\mu = \frac{10 + 14 + 35 + (3)(50) + 72 + 103}{8}$$

$$= \underline{\underline{48}}$$

$$\sigma^2 = E(x^2) - (\mu)^2$$

$$= \frac{(10)^2 + (14)^2 + (35)^2 + 3(50)^2 + (72)^2 + (103)^2}{8} - (48)^2$$

$$= 3101.75 - (48)^2 = \underline{\underline{797.75}}$$

$$\text{Coefficient of Variation} = \frac{\sigma}{\mu}$$

$$= \frac{\sqrt{797.75}}{48} = \underline{\underline{0.58843}}$$

9. Losses under an insurance policy based on US currency follows a **single parameter** Pareto distribution with $\alpha = 2$. The $VaR_p(X) = 10$ in US dollars. Each US dollar is worth 7.8 Hong Kong dollars.

Calculate the $TVaR_p(X)$ in Hong Kong dollars.

From the tables

$$VaR_p(x) = \theta(1-p)^{-\frac{1}{\alpha}} = \theta(1-p)^{-\frac{1}{2}} = 10 \text{ in USD}$$

$$TVaR_p(x) = \frac{\alpha\,\theta\,(1-p)^{-\frac{1}{\alpha}}}{\alpha-1} = \frac{2\theta\,(1-p)^{\frac{1}{2}}}{2-1} = 2\,\theta(1-p)^{\frac{1}{2}}$$

$\theta_{HKD} = 7.8\,\theta_{USD}$ because θ is scale parameter

$$TVaR_p(x) \text{ in HKD} = 2\,(7.8\,\theta_{USD})(1-p)^{\frac{1}{2}}$$

$$= (2)(7.8)\left[\theta\,(1-p)^{\frac{-1}{2}}\right]$$

$$= 2(7.8)\left[VaR_p(x)\right] = (2)(7.8)(10) = \underline{156}$$

10. You are given the following distribution of losses:

X	Probability
100	0.20
200	0.25
300	0.30
400	0.15
500	0.10

Calculate the Loss Elimination Ratio if an ordinary deductible of 250 is applied.

$$LER = \frac{E[X \wedge 250]}{E[X]}$$

$$E[X] = 100(.2) + 200(.25) + 300(.3) + 400(.15) + 500(.1)$$

$$= 270$$

$$E[X \wedge 250] = 100(.2) + 200(.25) + 250(1 - .2 - .25)$$

$$= 207.50$$

$$LER = \frac{207.50}{270} = 0.7685$$