STAT 479

Test 2

November 9, 2010

1. You are given the following grouped claim information:

Amount of Claim	Number of Claims	Total Amount of Claims
0 – 5000	8	24,000
5000 – 10,000	12	96,000
10,000 – 25,000	3	55,000
25,000 – 100,000	2	150,000
100,000 +	1	125,000
Total	26	450,000

Calculate $E(X \Lambda 10,000)$ and $E(X \Lambda 40,000)$.

$$E(X \wedge 10,000) = \frac{24,000 + 96000 + 6(10,000)}{26}$$

$$= 6923.08$$

$$E(X \wedge 40,000) = \frac{24000 + 96000 + 55000 + 3(40,000)^{*}}{26}$$

$$= 11,346.15$$

* This is true because all claims must be above 40,000. Clearly claim over 100,000 is above 40,000. For claims between 25,000 & 100,000, even if one claim is 100,000, the other is 50,000.

2. The number of dental insurance claims follow a Poisson distribution with an expected number of claims of 2.

The amount of each dental claim has the following distribution:

Amount of Claim	Probability of Claim
100	0.50
200	0.25
300	0.20
400	0.05

Dunham Dental Insurance Company has 2000 independent insureds with dental insurance.

Use the normal approximation to estimate the probability that aggregate claims will exceed 700,000 in a given year.

$$E(N) = \lambda = 2$$

$$E(X) = (.5)(100) + (.25)(200) + (.20)(300) + (400)(.05)$$

$$= 180$$

$$E(S) = E(N)(E(X)) = 360$$

$$E(X^2) = (.5)(100^2) + (.25)(200^2) + (.2)(300^2) + (.05)(400^2)$$

$$= 41{,}000$$

$$Var(S) = [E(X^2)](\lambda) = (41{,}000)(2) = 82{,}000$$

$$z = \frac{700{,}000 - 360(2000)}{\sqrt{2000(82{,}000)}} = -1.5617$$

$$Pr(z > -1.5617) = \underline{\underline{0.9406}}$$

3. The number of dental insurance claims follow a Poisson distribution with an expected number of claims of 2.

The amount of each dental claim has the following distribution:

Amount of Claim	Probability of Claim
100	0.50
200	0.25
300	0.20
400	0.05

Calculate $f_s(400)$ for a person with dental insurance.

400 $\Rightarrow$ 1 claim 400

2 claims 200 or 100 & 300

3 claims 100, 100, 200

4 claims 100, 100, 100, 100

$$Pr(N=1)\,Pr(X=400) + Pr(N=2)\left[(Pr(X=200))^2 + 2(Pr(X=100)\cdot Pr(X=300))\right] +$$

$$Pr(N=3)\cdot 3\cdot(Pr(X=100))^2\cdot Pr(X=200) +$$

$$Pr(N=4)\cdot(Pr(X=100))^4$$

$$= (e^{-2})(2)(.05) + \frac{e^{-2}2^2}{2!}\left[(.25)^2 + (2)(.5)(.2)\right]$$

$$+ \frac{e^{-2}2^3}{3!}\cdot 3\cdot(.5^2)(.25) + \frac{e^{-2}2^4}{4!}(.5)^4$$

$$= \underline{0.12406}$$

4. Warranty claims for laptop computers follow a Pareto distribution with $\alpha = 5$ and $\theta = 2000$.

Chengyin decides to discretize this distribution using a span of 200.

$f^R(600)$ is the probability assigned to the value of 500 using the Method of Rounding.

$f^{MM}(600)$ is the probability assigned to the value of 500 using the Method of Moment Matching.

Calculate $1000\{f^R(600) - f^{MM}(600)\}$.

$$f^R(600) = F(700) - F(500) =$$

$$1 - \left(\frac{2000}{2700}\right)^5 - \left[1 - \left(\frac{2000}{2500}\right)^5\right] = 0.10467$$

$$f^{mm}(600) = \frac{2E[X \wedge 600] - E(X \wedge 400) + E(X \wedge 800)}{200}$$

$$= \frac{2\left\{\frac{2000}{4}\left[1 - \left(\frac{2000}{2600}\right)^4\right]\right\} - \frac{2000}{4}\left[1 - \left(\frac{2000}{2400}\right)^4\right] - \frac{2000}{4}\left[1 - \left(\frac{2000}{2800}\right)^4\right]}{200}$$

$$= .10576$$

$$1000(0.10467 - 0.10576) = \underline{\underline{-1.09}}$$

5. The following information on students in the actuarial program at Purdue is used to complete an analysis of students leaving the program because they are switching majors.

Note 4 lives

Student	Time of Entry	Time of Exit	Reason for Exit
1	0	.5	Switching Major
2-5	0	1	Switching Major
6	0	2	Switching Major
7	0	3	Graduation
8	0	3	Switching Major
9-12	0	3.5	Graduation
13-23	0	4	Graduation
24	0.5	2	Switching Major
25	0.5	3	Switching Major
26	1	3.5	Graduation
27	1	4	Switching Major
28	1.5	4	Graduation
29	2	5	Graduation
30	3	5	Graduation

$\hat{H}(3)$ is estimated using the Nelson-Åalen estimator.

Calculate the 90% linear confidence interval for $\hat{H}(3)$.

j	x	s	r	$\hat{H}(x)$
1	.5	1	23	
2	1	4	24	
3	2	2	23	
4	3	2	22	$1/23 + 4/24 + 2/23 + 2/22$

$= 0.38801$

$$Var = \frac{1}{23^2} + \frac{4}{24^2} + \frac{2}{23^2} + \frac{2}{22^2} = 0.016747753$$

Linear confidence interval =

$$0.38801 \pm 1.645\sqrt{0.016747753}$$

$(\cancel{.23860\ ,\ .51742})$ 0.175125; 600895

6. The following information on students in the actuarial program at Purdue is used to complete an analysis of students leaving the program because they are switching majors.

Student	Time of Entry	Time of Exit	Reason for Exit
1	0	.5	Switching Major
2-5	0	1	Switching Major
6	0	2	Switching Major
7	0	3	Graduation
8	0	3	Switching Major
9-12	0	3.5	Graduation
13-23	0	4	Graduation
24	0.5	2	Switching Major
25	0.5	3	Switching Major
26	1	3.5	Graduation
27	1	4	Switching Major
28	1.5	4	Graduation
29	2	5	Graduation
30	3	5	Graduation

$\hat{S}(x)$ is estimated using the product limit estimator.

Estimate $Var[S_{30}(2)]$ using the Greenwood approximation.

$$Var\left[S_{30}(2)\right] \doteq \left[\hat{S}(2)\right]^2 \left[\sum \frac{s_i}{(r_i)(r_i - s_i)}\right]$$

$$\hat{S}(2) = \left(\frac{22}{23}\right)\left(\frac{20}{24}\right)\left(\frac{21}{23}\right) = 0.72779$$

$$Var = (.72779)^2 \left[\frac{1}{(23)(22)} + \frac{4}{(24)(20)} + \frac{2}{(21)(23)}\right]$$

$$= 0.007654$$

7. Ab Ghani Automobile Insurance Company received the following claims under an automobile insurance policy:

100 100 200 200 250 300 300 300 400 700

Calculate $F_{10}(200)$ using the Empirical Distribution Function. $= \frac{\# \leq 200}{\text{TOTAL}} = \frac{4}{10} = 0.4$

Calculate $\hat{F}(200)$ if the distribution is smoothed using the **uniform** kernel with a bandwidth of 100.

Y	P_Y	Range	$K_y(x)$
100	$2/10$	0-200	1 since at top of range
200	$2/10$	100-300	$\frac{x-200+100}{200}$
250	$1/10$	150-350	$\frac{x-250+100}{200}$
300 ↓	$5/10$	200+	0 since at or below range

$$\hat{F}(200) = \tfrac{2}{10}(1) + \tfrac{2}{10}\left(\frac{200-200+100}{200}\right) + \tfrac{1}{10}\left(\frac{200-250+100}{200}\right)$$

$$= .2 + .2\left(\tfrac{1}{2}\right) + .1\left(\tfrac{1}{4}\right) = 0.325$$

8. Ab Ghani Automobile Insurance Company received the following claims under an automobile insurance policy:

100 100 200 200 250 300 300 300 400 700

$\sum X = 2850$ and $\sum X^2 = 1,082,500$

The company's Chief Actuary, Rahim, creates a continuous distribution using a Kernel Density model with a **triangular** kernel with a bandwith of 100. Rahim then uses this Kernel Density model to calculate the premium to be charged. The premium is calculated as the mean plus one standard deviation.

Calculate the premium.

FOR EMPIRICAL

$$E(X) = \frac{2850}{10} = 285$$

$$Var(X) = E(X^2) - (E(X))^2 = \frac{1,082,500}{10} - (285)^2$$

$$= 27025$$

For Triangular Kernal

$$E(X) = E(X)^{EMPIRICAL} = 285$$

$$Var(X) = Var(X)^{EMPIRICAL} + \frac{b^2}{6} = 28691.6\bar{6}$$

$$PREMIUM = E(X) + \sqrt{Var(X)}$$

$$= 285 + \sqrt{28,691.6\bar{6}}$$

$$= 454.386$$

9. You are given:

   i. The frequency distribution for claims is distributed as a geometric distribution with $\beta = 2$.

   ii. The severity distribution for claims is distributed as follows:

Amount of Claim	Probability
400	0.5
800	0.4
1000	0.1

Sutton Stop Loss LTD provides stop loss coverage with an aggregate deductible of 1000.

Calculate the net stop loss premium.

$$E[(S-1000)_+] = E(S) - E(S \wedge 1000)$$

$$E(S) = E(N)\,E(X) = (2)(0.5\cdot 400 + 0.4\cdot 800 + 0.1\cdot 1000)$$

$$= 1240$$

$$f(0) = p_0 = \frac{1}{3}$$

$$f(400) = p_1\,(Pr(X=400)) = (2/9)(.5) = \frac{1}{9}$$

$$f(800) = p_1\,(Pr(X=800)) + p_2\,(Pr(X=400))^2$$

$$= 2/9(.4) + \frac{4}{27}(.5)^2 = 0.125926$$

$$E(S \wedge 1000) = 1/3(0) + \frac{1}{9}(400) + .125926(800) + 1000(1 - 1/3 - 1/9 - 0.125926)$$

$$= 574.815$$

$$E[(S-1000)_+] = 1240 - 574.815 = 665.185$$

10. You are given the following sample:

X: 10 20 30 40 50

The following estimator is used to estimate σ^2:

$$\frac{\sum_i (X_i - \bar{X})^2}{n+1}$$

Calculate the bias in this estimator.

unbiased Estimator of σ^2 $= \dfrac{\sum (x_i - \bar{x})^2}{N-1}$

bias $= E(\hat{\sigma}^2 | \sigma^2) - \sigma^2$

$$= E\left(\frac{\sum (x_i - \bar{x})^2}{n+1}\right) - \sigma^2$$

$$= \frac{n-1}{n+1} E\left(\frac{\sum (x_i - \bar{x})^2}{n-1}\right) - \sigma^2$$

$$= \frac{n-1}{n+1} \sigma^2 - \sigma^2$$

$$= -\frac{2}{n+1} \sigma^2$$

but $n = 5$

so $= -\frac{1}{3} \sigma^2$