1. Purdue Life Insurance Company (PLIC) wants to develop a multiple decrement model to use in pricing a two year term insurance policy. The multiple decrement model will have two decrements – (d) death and (w) lapse.

   However, PLIC does not have sufficient data to develop a multiple decrement table. Therefore, PLIC will use an independent mortality table and an independent lapse table to develop a multiple decrement table.

   You are given the following independent mortality and lapse tables:

x	$q_x'^{(d)}$		x	$q_x'^{(w)}$
80	0.08		80	0.20
81	0.12		81	0.06

   In deriving the multiple decrement table, you assume that decrements are uniformly distributed in each independent single decrement table.

   a. (6 points) Complete the following multiple decrement table. Be sure to show your work:

x	$l_x^{(\tau)}$	$d_x^{(d)}$	$d_x^{(w)}$
80	10,000		
81			

   PLIC wants to develop the premium for a fully discrete two year term insurance policy on (80). The death benefit is 100,000. If the policy is surrendered (withdrawal) in the first year, a benefit equal to 50% of one annual premium is paid at the end of the year. If the policy is surrendered after the first year, there is no benefit paid on withdrawal.

   b. (8 points) Using the above multiple decrement table and $i = 0.05$, calculate the net benefit premium.

   c. (6 points) Calculate the reserve for this policy at the end of year 1.

   Let L_0 be the loss at issue random variable for this policy.

   d. (Bonus) Calculate $Var(L_0)$.

Part a.

Given that decrements are uniformly distributed in the **Single Decrement Table.**

$$q_{80}^{(d)} = q_{80}'^{(d)}(1-0.5q_{80}'^{(w)}) = (0.08)[1-(0.5)(0.20)] = 0.072$$

$$q_{80}^{(w)} = q_{80}'^{(w)}(1-0.5q_{80}'^{(d)}) = (0.20)[1-(0.5)(0.08)] = 0.192$$

$$q_{81}^{(d)} = q_{81}'^{(d)}(1-0.5q_{81}'^{(w)}) = (0.12)[1-(0.5)(0.06)] = 0.1164$$

$$q_{81}^{(w)} = q_{81}'^{(w)}(1-0.5q_{81}'^{(d)}) = (0.06)[1-(0.5)(0.12)] = 0.0564$$

x	$l_x^{(\tau)}$	$d_x^{(d)}$	$d_x^{(w)}$
80	10,000	(10,000)(0.072)=720	(10,000)(0.192)=1920
81	10,000-720-1920=7360	(7360)(0.1164)=856.704	(7360)(0.0564)=415.104

Part b and c

$$P[10,000+7360v] = 100,000[720v+856.704v^2]+0.5P[1920v]$$

$$17,009.52381P = 146,277,006.8+914.2857P$$

$$P = 9088.22$$

$${}_0V = 0$$

$${}_1V = \frac{(0+9088.22)(1.05)-(100,000)(0.072)-(0.5)(9088.22)(0.192)}{1-0.072-0.192} = 1997.50$$

or

$$PVFB - PVFP = 100,000(0.1164)v - 9088.22 = 1997.49$$

Part d.

This has to be done from first principles. You cannot use any of the shortcut formulas.

Possible loss events are listed below. There are four of these. The probability of each loss event is also listed. They should total to 1.

$100,000v - 9088.22$ is the loss if die in year 1 Prob=0.072

(0.5)(9088.22)v-9088.22 is the loss if withdraw in year 1 Prob=0.192

$100{,}000v^2 - 9088.22 - 9088.22v$ is the loss if die in year 2 Prob=(0.736)(0.1164)

$-9088.22 - 9088.22v$ is the loss if do not die in year 2 Prob=$(0.736)(1-0.1164)$

$Var = E[(L_0)^2]$ since $E[(L_0)] = 0$ due to equivalence principle

$$= (100,000v - 9088.22)^2(0.072) + ((0.5)(9088.22)\text{v-}9088.22)^2(0.192) +$$
$$(100{,}000v^2 - 9088.22 - 9088.22v)^2(0.736)(0.1164) + (-9088.22 - 9088.22v)^2(0.736)(1-0.1164)$$

$1{,}199{,}497{,}526$