1. A two-year temporary life annuity due provides a payment of 1000 at time 0 and another payment of 1000 at time 1 if (x) is alive.

You are given:

i. \( v = 0.95 \)

ii. Mortality is a random variable with the following distribution:

<table>
<thead>
<tr>
<th>Mortality Rate for (x)</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.08</td>
<td>0.30</td>
</tr>
<tr>
<td>0.10</td>
<td>0.60</td>
</tr>
<tr>
<td>0.12</td>
<td>0.10</td>
</tr>
</tbody>
</table>

iii. \( Y \) is the present value random variable for this annuity.

a. (7 points) Calculate the \( E[Y] \).

Solution:

\[
P V(q_x = 0.08) = 1000 + 1000(0.95)(1 - 0.08) = 1874
\]

\[
P V(q_x = 0.10) = 1000 + 1000(0.95)(1 - 0.10) = 1855
\]

\[
P V(q_x = 0.12) = 1000 + 1000(0.95)(1 - 0.12) = 1836
\]

\[
E P V = (0.3)(1874) + (0.6)(1855) + (0.1)(1836) = 1858.80
\]
b. (13 points) Calculate the $Var[Y]$.

**Solution:**

$PV(q_x = 0.08 & Die) = 1000 \Rightarrow Pr = (0.3)(0.08) = 0.024$

$PV(q_x = 0.08 & Live) = 1000 + 1000(0.95) = 1950 \Rightarrow Pr = (0.3)(0.92) = 0.276$

$PV(q_x = 0.10 & Die) = 1000 \Rightarrow Pr = (0.6)(0.1) = 0.06$

$PV(q_x = 0.10 & Live) = 1000 + 1000(0.95) = 1950 \Rightarrow Pr = (0.6)(0.90) = 0.54$

$PV(q_x = 0.12 & Die) = 1000 \Rightarrow Pr = (0.1)(0.12) = 0.012$

$PV(q_x = 0.12 & Live) = 1000 + 1000(0.95) = 1950 \Rightarrow Pr = (0.1)(0.88) = 0.088$

Now regroup

$PV=1000$ with $Pr = 0.096$ and $PV=1950$ with $Pr = 0.904$

$Var[Y] = E[Y^2] - (E[Y])^2 = (1000)^2(0.096) + (1950)^2(0.904) - (1858.8)^2 = 78,322.56$