STAT 490
Test 1
Spring 2015
February 18, 2015

1. (30 points) Christian has started to work today at Spears Corporation. Today is Christian's 42nd birthday and his annual salary at his new job is 115,000.

   Spears Corporation provides both a Defined Benefit plan and a Defined Contribution plan. The Defined Benefit Plan provides an annual retirement benefit that is 1.6% of the final three year average salary for each year of service.

   Christian plans to retire at age 65 and wants a total replacement ratio of 75%. In order to achieve this goal, Christian intends to make annual contributions to the Defined Contribution plan. Christian's contributions will be made on each birthday beginning with his 43rd birthday. The last contribution will be made on his 65th birthday. Each contribution will be $X\%$ of his salary during the last year.

   Christian determines his contribution based on the following assumptions:

   i. The Defined Contribution plan will earn 8% prior to retirement.
   ii. Christian's salary will increase by 3% on each birthday beginning with his 43rd birthday.
   iii. The annuity that will be purchased by the Defined Contribution plan will be a survivor annuity. Assume that the annuity is priced using the Illustrative Life Table with a 6% interest rate and assuming both lives are the same age.

   a. (10 points) Christian's annual retirement benefit provided by the Defined Benefit plan is approximately 79,000 to the nearest 1000. Calculate it to the nearest 1.

      Solution:

$$0.016 * \frac{115,000(1.03)^{62-42}(1+1.03+1.03^2)}{3}(65-42) = 78,751$$

b. (5 points) Calculate the benefit needed under the Defined Contribution plan to achieve his desired total replacement ratio.

Solution:

$$\frac{TotalPension}{115,000(1.03)^{22}} = 0.75 \implies TotalPension = 165,263.919$$

$$DefinedBenefit = 78,751$$

$$DefinedContribution = TotalPension - DefinedControbution =$$

$$165,263.92 - 78,751 = 86,513$$

c. (5 points) Determine the amount that Christian needs at age 65 to fund the benefit from the Defined Contribution plan.

Solution:

$$(DefinedContribution)\ddot{a}_{\overline{65:65}} = (86,513)(2\ddot{a}_{65} - \ddot{a}_{65:65}) =$$

$$(86,513)(2*9.8969 - 7.8552) = 1,032,844$$

d. (10 points) Determine X .

Solution:

$$(X)(115,000)[1.08^{22} + (1.03)(1.08)^{21} + ... + (1.03)^{22}(1.08)^{0}]$$

$$= (X)(115,000)\frac{(1.08)^{22} - (1.03)^{23}(1.08)^{-1}}{1 - \frac{1.03}{1.08}} = 1,032,844$$

$$X = 11.52\%$$

2. (10 points) A survivor whole life policy on (50) and (40) has a death benefit of 1 million payable at the moment of death. The two lives are assumed to be independent. Premiums are paid annually.

You are given:

i. Mortality follows the Illustrative Life Table.
ii. Deaths are uniformly distributed between integral ages.
iii. Interest is 6%.

P is the net annual benefit premium assuming that premiums are paid as long as one of the insureds is alive.

Calculate P.

Solution:

$$P(\ddot{a}_{\overline{40:50}}) = 1{,}000{,}000(\bar{A}_{\overline{40:50}})$$

$$\bar{A}_{\overline{40:50}} = (\frac{i}{\partial})\left[A_{40} + A_{50} - A_{40:50}\right] = (\frac{.06}{\ln(1.06)})\left[.16132 + .24905 - .29368\right] = .1201567$$

$$\ddot{a}_{\overline{40:50}} = \ddot{a}_{40} + \ddot{a}_{50} - \ddot{a}_{40:50} = 14.8166 + 13.2668 - 12.4784 = 15.605$$

$$P = \frac{(1{,}000{,}000)(.1201567)}{15.605} = 7{,}699.88$$

3. (10 points) Jeff, (59), and Jean, (49) have 800,000 in a fund and are going to purchase an annuity with this money. They have the option of the following two annuities:

   a. Annuity 1 is an annuity due with annual payments of B if both of them are alive. It pays $0.5B$ if Jeff is alive and Jean is dead and $0.6B$ if Jean is alive and Jeff is dead.

   b. Annuity 2 is a reversionary annuity due with annual payments of R to Jean after the death of Jeff.

   Assume that the lifetimes are independent. Mortality follows the Illustrative Life Table and interest is at 6%.

   Calculate $R-B$.

   Solution:

$$\text{Annuity}1:$$

$$a = 0.5B \quad b = 0.6B \quad c = B - a - b = -0.1B$$

$$a\ddot{a}_{59} + b\ddot{a}_{49} + c\ddot{a}_{49:59} = 0.5B*11.3818 + 0.6B*13.4475 - 0.1B*10.4441 = 800,000$$

$$B = 62,917.86$$

$$Annuity2:$$

$$R\ddot{a}_{59|49} = R(\ddot{a}_{49} - \ddot{a}_{59:49}) = R(13.4475 - 10.4441) = 800,000$$

$$R = 266,364.79$$

$$R - B = 266,364.79 - 62,917.86 = 203,446.93$$

4. (10 points) A male age 75 and a female age 65 are subject a common shock model.

You are given:

i. Male and female mortality follows the Illustrative Life Table.
ii. The common shock risk is included in the above mortality rates.
iii. Both lives are subject to a common shock based on a constant force of mortality of 0.015.

Calculate ${}_{10}p_{\overline{75:65}}$.

Solution:

$${}_{10}p_{\overline{65:75}} = {}_{10}p_{65} + {}_{10}p_{75} - {}_{10}p_{65:75}$$

$$= \frac{l_{75}}{l_{65}} + \frac{l_{85}}{l_{75}} - \left(\frac{l_{75}}{l_{65}}\right)\left(\frac{l_{85}}{l_{75}}\right)e^{(0.015)(10)} =$$

$$\frac{5,396,081}{7,533,964} + \frac{2,358,246}{5,396,081} - \left(\frac{5,396,081}{7,533,964}\right)\left(\frac{2,358,246}{5,396,081}\right)e^{(0.15)} = 0.78959$$

5. (10 points) The Jiang Life Insurance Company has 100,000 independent lives all age x.

Jana models these lives in a multiple decrement model using rates derived from single decrement tables. The decrements are (d) death and (w) withdrawal. The decrements between integral ages are assumed to be uniformly distributed in the multiple decrement model.

You are given:

i. $p_x'^{(d)} = 0.9$

ii. $p_x'^{(w)} = 0.82$

Let D be the random variable representing the number of deaths between age x and $x+1$.

Calculate the $Var[D]$.

Solution:

$$p_x^{(\tau)} = p_x'^{(d)} \cdot p_x'^{(w)} = (0.9)(0.82) = 0.738$$

$$p_x'^{(d)} = (p_x^{(\tau)})^{\frac{q_x^{(d)}}{q_x^{(\tau)}}} \implies 0.9 = (0.738)^{\frac{q_x^{(d)}}{1-0.738}}$$

$$\implies q_x^{(d)} = (1-0.738)\left(\frac{\ln(0.9)}{\ln(0.738)}\right) = 0.09086$$

$$E[D] = nq_x^{(d)} = (100,000)(0.09086) = 9086$$

$$Var[D] = nq_x^{(d)}(1-q_x^{(d)}) = (100,000)(0.09086)(1-0.09086) = 8260$$

6. (30 points) Purdue Life Insurance Company (PLIC) wants to develop a multiple decrement model to use in pricing a special life annuity. The multiple decrement model will have two decrements – (d) death and (r) remarriage.

However, PLIC does not have sufficient data to develop a multiple decrement table. Therefore, PLIC will use an independent mortality table and an independent remarriage table to develop a multiple decrement table.

You are given the following independent mortality and lapse tables:

x	$q_x'^{(d)}$		x	$q_x'^{(r)}$
90	0.30		90	0.40
91	0.50		91	0.20

In deriving the multiple decrement table, you assume that decrements are uniformly distributed in each independent single decrement table.

a. (10 points) Complete the following double decrement table. Be sure to show your work:

x	$l_x^{(\tau)}$	$d_x^{(d)}$	$d_x^{(r)}$
90	100,000		
91			

Solution:

$$q_x^{(d)} = q_x'^{(d)}(1-0.5q_x'^{(r)})$$

$$q_{90}^{(d)} = q_{90}'^{(d)}(1-.5q_{90}'^{(r)}) = 0.3(1-0.5(0.4)) = 0.24$$
$$q_{90}^{(r)} = q_{90}'^{(r)}(1-0.5q_{90}'^{(d)}) = 0.4(1-0.5(0.3)) = .34$$
$$q_{91}^{(d)} = q_{91}'^{(d)}(1-0.5q_{91}'^{(r)}) = .5(1-0.5(0.2)) = 0.45$$
$$q_{91}^{(r)} = q_{91}'^{(r)}(1-0.5q_{91}'^{(d)}) = .2(1-0.5(0.5)) = 0.15$$

$$d_{90}^{(d)} = (l_{90})(q_{90}^{(d)}) = (100,000)(0.24) = 24,000$$
$$d_{90}^{(r)} = (l_{90})(q_{90}^{(r)}) = (100,000)(0.34) = 34,000$$
$$d_{91}^{(d)} = (l_{91})(q_{91}^{(d)}) = (42,000)(0.45) = 18,900$$
$$d_{91}^{(r)} = (l_{91})(q_{91}^{(r)}) = (42,000)(0.15) = 6,300$$

A special **two-year temporary** life annuity due to (90) makes an annual payment of 100,000 at the beginning of each year provided that the annuitant has not died or remarried. In other words, payments stop upon either remarriage or death. In addition, the annuity pays a lump sum if the annuitant gets remarried. The lump sum is 25,000 paid at the end of the year of remarriage.

b. (10 points) Using the above multiple decrement table and $i = 0.05$, calculate the actuarial present value of this annuity.

Solution:

$$APV = 100{,}000 + 100{,}000(p_{90}^{(\tau)})(v) + 25{,}000\left[(q_{90}^{(r)})(v) + (p_{90}^{(\tau)})(q_{91}^{(r)})(v^2)\right]$$

$$= 100{,}000\left[1 + 0.42\left(\frac{1}{1.05}\right)\right] + 25{,}000\left[(0.34)(\frac{1}{1.05}) + (0.42)(0.15)(\frac{1}{1.05})^2\right]$$

$$= 140{,}000 + 9{,}523.81 = 149{,}523.81$$

c. (10 points) Let Y be the present value at issue random variable for this annuity. Y can take on four possible values. Calculate each of those four possible values and assign a probability to each one.

Solution:

4 Possible Scenarios:
-Insured dies in 1st year
-Insured gets remarried in 1st year
-Insured does not get remarried in 2nd year
-Insured gets remarried in 2nd year

$$PV(dies1st) = 100{,}000$$

$$PV(remarries1st) = 100{,}000 + 25{,}000(\frac{1}{1.05}) = 123{,}809.52$$

$$PV(DoesNot\,\mathrm{Re}marry2nd) = 100{,}000\left[1 + (\frac{1}{1.05})\right] = 195{,}238.10$$

$$PV(remarries2nd) = 100{,}000\left[1 + (\frac{1}{1.05})\right] + 25{,}000(\frac{1}{1.05})^2 = 217{,}913.83$$

Pr(D1st) = 0.24
Pr(R1st) = 0.34
Pr(DNR2nd) = (0.42)(0.85) = 0.357
Pr(R2nd) = (0.42)(0.15) = 0.063