1. A participating whole life insurance policy issued on (50). The policy pays a death benefit of 75,000 at the end of the year of death. You are given:

- Reserves are determined as net premium reserves using the Full Preliminary Term method. The mortality follows the Illustrative Life Table and \( i = 0.06 \).
- The annual gross premium is 130% of the second year net premium under Full Preliminary Term.
- Actual mortality experienced is 95% of the Illustrative Life Table.
- Actual interest earned is 6.8%
- Expenses for the policy are:
  - Issue expenses of 200 per policy and 34% of premium.
  - Recurring expenses are 30 per policy and 6% of premium in all years including the first year.
- Cash values are 85% of the reserves.
- Withdrawals will occur at the end of the year. The rate of withdrawals will be 12% in the first year, 8% in the second year, and 6% per year thereafter.
- 80% of the profit each year will be distributed to the policyholders who were in force at the start of the year.

a. Calculate the dividend in for the 10th year.
Solution:

Profit = \((\text{BegReserve} + \text{Premium} - \text{Expense})(1+i) - \text{Benefits} - (\text{EndReserve})p_{(r)}\)

\[ P_{\text{FPT}}^{\text{FPT}} = \frac{75,000A_{51}}{\ddot{a}_{51}} = \frac{(75,000)(0.25961)}{13.0803} = 1488.555308 \]

\[ P^{\text{Gross}} = (1.3)(1488.555308) = 1935.12 \]

\[ \nu V = 75,000 \left(1 - \frac{\ddot{a}_{59}}{\ddot{a}_{51}}\right) = 75,000 \left(1 - \frac{11.3818}{13.0803}\right) = 9738.88 \]

\[ i_{10} V = 75,000 \left(1 - \frac{\ddot{a}_{60}}{\ddot{a}_{51}}\right) = 75,000 \left(1 - \frac{11.1454}{13.0803}\right) = 11,094.36 \]

\[ i_{10} CV = (11,094.36)(0.85) = 9430.21 \]

Profit = \((9738.88 + (1935.12)(0.94) - 30)(1.068) - (75,000)(0.01262)(0.95) - (9430.21)(1 - (0.01262))(0.06) - (11,094.36)(1 - (0.01262)(0.95))(1 - 0.06)\)

= 549.92

\[ D_{\text{Div}} = (0.8)(549.92) = 439.93 \]

b. If the dividend is only paid to those who are in force (have not died or withdrawn) at the end of the year, determine the dividend per policy.

Solution:

\[ \frac{\text{(Dividend from Part (a))}}{P_{59}^{(r)}} = \frac{439.93}{(0.94)(1 - (0.95)(0.01262))} = 473.69 \]
2. You are given the following information for a Type B universal life contract:

<table>
<thead>
<tr>
<th>Policy Year</th>
<th>Annual Premium</th>
<th>Percent of Premium Charge</th>
<th>Annual Expense Charge</th>
<th>Additional Death Benefit</th>
<th>Annual Cost of Insurance Charge</th>
<th>Annual Discount Rate for COI</th>
<th>Annual Credited Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3000</td>
<td>20%</td>
<td>30</td>
<td>100,000</td>
<td>0.010</td>
<td>4%</td>
<td>5%</td>
</tr>
<tr>
<td>2</td>
<td>2500</td>
<td>8%</td>
<td>30</td>
<td>100,000</td>
<td>0.015</td>
<td>4%</td>
<td>5.2%</td>
</tr>
</tbody>
</table>

Calculate the account value at the end of one year and at the end of two years.

**Solution:**

\[
AV_t = (AV_{t-1} + P_t(1-f) - EC_t - ADB_t \cdot q^{COI}_{x+t-1})(1+i_t)
\]

\[
AV_1 = \left[ 0 + 3000(1-0.2) - 30 - \left( \frac{100,000}{1.04} \right)(0.01) \right](1.05) = 1478.88
\]

\[
AV_2 = \left[ 1478.88 + 2500(1-0.08) - 30 - \left( \frac{100,000}{1.04} \right)(0.015) \right](1.052) = 2426.51
\]
3. For a Type B universal life contract on (40) with an Additional Death Benefit of 200,000, you are given:

   i. The annual premium is 4000 at the start of each year.
   ii. The percent of premium charges are 60% in the first year and 10% thereafter.
   iii. The per policy expense charge is 100 in the first year and $X$ in years 2 and later.
   iv. The annual cost of insurance is 90% of the mortality rate in the Illustrative Life Table.
   v. The credited interest rate is 6%.
   vi. The account value at the end of three years is 8734.02.

Determine $X$.

**Solution:**

\[
AV_t = (AV_{t-1} + P_t(1 - f) - EC_t - ADB_t q_x^{COL} c x_{t-1}) (1 + i_t^c)
\]

\[
AV_1 = \left[ 0 + 4000(1 - 0.6) - 100 - \left( \frac{100,000}{1.06} \right)(0.90)(0.00278) \right] (1.06) = 1339.80
\]

\[
AV_2 = \left[ 1339.80 + 4000(1 - 0.10) - X - \left( \frac{100,000}{1.06} \right)(0.90)(0.00298) \right] (1.06) = 4967.988 - 1.06X
\]

\[
AV_3 = \left[ 4967.988 - 1.06X + 4000(1 - 0.10) - X - \left( \frac{100,000}{1.06} \right)(0.90)(0.00320) \right] (1.06)
\]

\[
= 8794.06728 - 2.1836X = 8730.02 \implies X = \frac{8794.06728 - 8734.02}{2.1836} = 27.50
\]
4. Xintong purchased a Type B universal life contract. If she pays a premium of 1000 at the beginning of each year for 20 years, she will have an account value of 18,000 at the end of 20 years.

The universal life policy has the following characteristics:

   i. The percent of premium charge for all years is 8.5%.
   ii. The credited interest rate is 6% in all years.

Instead of paying 1000 in all years, Xintong decides to pay 2000 in years 1 and 2 and then pay 600 at the beginning of years 3 through 20.

Calculate Xintong’s account value after 20 years with this revised premium pattern.
Solution:

\[ AV_1 = (AV_{t-1} + P_t(1 - f) - EC_t - ADB_t, q_{x+t+1}^{COI})(1 + i_t) \]

\[ = (AV_{t-1} + P_t(1 - 0.085) - EC_t - \frac{ADB_t \cdot q_{x+t+1}^{COI}}{1.06})(1.06) = (AV_{t-1} + P_t(0.915) - EC_t)(1.06) - ADB_t \cdot q_{x+t+1}^{COI} \]

\[ AV_1 = 0 + (0.915)P_t(1.06) - EC_t(1.06) - ADB_t \cdot q_{x+t+1}^{COI} \]

\[ AV_2 = [(0.915)P_t(1.06) - EC_t(1.06) - ADB_t \cdot q_{x+t+1}^{COI} + P_{2} - EC_{2}](1.06) - ADB_{2} \cdot q_{x+1}^{COI} = \]

\[ 0.915 \sum_{k=1}^{3} P_k (1.06)^{3-k} - \sum_{k=1}^{3} EC_k (1.06)^{3-k} - \sum_{k=1}^{3} ADB_k (1.06)^{2-k} q_{x+k-1}^{COI} \]

By Induction

\[ AV_t = 0.915 \sum_{k=1}^{t} P_k (1.06)^{t+1-k} - \sum_{k=1}^{t} EC_k (1.06)^{t+1-k} - \sum_{k=1}^{t} ADB_k (1.06)^{t-k} q_{x+k-1}^{COI} \]

Note that the second and third term are not a function of the premium so they will not be effected by the premium payment pattern.

Under premium payment pattern 1, we have:

\[ AV_{20} = 18,000 = 0.915 \sum_{k=1}^{20} 1000(1.06)^{20+1-k} - \sum_{k=1}^{20} EC_k (1.06)^{20+1-k} - \sum_{k=1}^{20} ADB_k (1.06)^{20-k} q_{x+k-1}^{COI} \]

Under premium payment pattern 2, we have

\[ AV_{20} = 0.915 \sum_{k=1}^{20} (1000 - 400)(1.06)^{20+1-k} + (0.915)(1400)[(1.06)^{20} + (1.06)^{19}] \]

\[ - \sum_{k=1}^{20} EC_k (1.06)^{20+1-k} - \sum_{k=1}^{20} ADB_k (1.06)^{20-k} q_{x+k-1}^{COI} \]

\[ = 18,000 - 0.915 \sum_{k=1}^{20} (400)(1.06)^{20+1-k} + (0.915)(1400)[(1.06)^{20} + (1.06)^{19}] = \]

\[ 18,000 - 0.915(400)\bar{s}_{20} + (0.915)(1400)[1.06^{20} + 1.06^{19}] = 11,712.80 \]
5. You are given the following information for a Type A universal life contract:

<table>
<thead>
<tr>
<th>Policy Year</th>
<th>Annual Premium</th>
<th>Percent of Premium Charge</th>
<th>Annual Expense Charge</th>
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</tr>
</tbody>
</table>

Calculate the account value at the end of one year and at the end of two years.

Solution:

\[
COI_1 = \frac{(0.010)(1.04)^{-1}[100,000 - (0 + 3000(1 - 0.2) - 30)(1.05)]}{1 - (0.010)(1.04)^{-1}(1.05)} = 947.17
\]

\[
AV_1 = (0 + (3000)(1 - 0.2) - 30 - 947.17)(1.05) = 1493.97
\]

\[
COI_2 = \frac{(0.015)(1.04)^{-1}[100,000 - (1493.97 + 2500(1 - 0.08) - 30)(1.052)]}{1 - (0.015)(1.04)^{-1}(1.052)} = 1406.50
\]

\[
AV_2 = (1493.97 + (2500)(1 - 0.08) - 30 - 1406.50)(1.052) = 2480.06
\]
6. For a Type A universal life contract on (40) with a total death benefit of 40,000, you are given:
   
   i. The annual premium is 4000 at the start of each year.
   
   ii. The percent of premium charges are 60% in the first year and 10% thereafter.
   
   iii. The per policy expense charge is 100 in the first year and 30 in years 2 and later.
   
   iv. The annual cost of insurance is 90% of the mortality rate in the Illustrative Life Table.
   
   v. The credited interest rate is 6%.
   
   vi. The account value at the end of three years is 20,000.
   
   vii. The corridor factor in the 4th year is 1.95

Determine the account value at the end of the fourth year.

Solution:

\[
COI_4^f = \frac{(0.9)(0.00344)(1.06)^{-1}[40,000 - (20,000 + 4000(1 - 0.1) - 30)(1.06)]}{1 - (0.9)(0.00344)(1.06)^{-1}(1.06)} = 43.86
\]

\[
COI_4^c = \frac{(0.9)(0.00344)(1.06)^{-1}(1.95 - 1)((20,000 + 4000(1 - 0.1) - 30))(1.06)}{1 + (0.9)(0.00344)(1.06)^{-1}(1.06)(1.95 - 1)} = 69.12
\]

COI is the greater of the two so it is 69.12.

\[
AV_4 = (20,000 + (4000)(1 - 0.1) - 30 - 69.12)(1.06) = 24,910.93
\]