1. Haotian is a new employee at Li Enterprises. Haotian is exact age 30 and his salary during the next year will be 58,000. Haotian is a participant in a defined contribution plan which pays 2% of the 3 year final average salary for each year of service.

You are given:

a. Employees at Li Enterprises will receive a 5% salary increase on each birthday.

b. Haotian will retire at exact age 65.

c. The defined benefit plan pays an annual benefit at the beginning of each year. The first 10 payments are guaranteed. Payments thereafter continue for the life of the employee.

d. The basis for purchasing the life and ten year certain annuity will be the Illustrative Life Table with interest at 6%.

Haotian’s retirement benefit at age 65 will be 203,000 to the nearest 1000. Calculate it to the nearest 1.

Solution:

\[
Benefit = (35)(0.02) \left[ \frac{(1.05)^{34} + (1.05)^{33} + (1.05)^{32}}{3} \right] (58,000) = 203,291
\]

Calculate the amount that Li Enterprises will need to have when Haotian in 65 in order to buy the annuity that will pay his retirement benefit.

Solution:

\[
Amount = (203,291)(\ddot{a}_{65:10}) = (203,291)(\dddot{a}_{10} +_{10} E_{65} \cdot \ddot{a}_{75}) = (203,291)[7.80169 + (0.39994)(7.2170)] = 2,172,786
\]
2. You are given the following information for a Type B universal life policy:

<table>
<thead>
<tr>
<th>Policy Year</th>
<th>Annual Premium</th>
<th>Percent of Premium Charge</th>
<th>Annual Expense Charge</th>
<th>Additional Death Benefit</th>
<th>Annual Cost of Insurance Charge</th>
<th>Annual Discount Rate for COI</th>
<th>Annual Credited Interest Rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10,000</td>
<td>25%</td>
<td>60</td>
<td>100,000</td>
<td>0.02</td>
<td>4%</td>
<td>6%</td>
</tr>
<tr>
<td>2</td>
<td>12,500</td>
<td>10%</td>
<td>30</td>
<td>100,000</td>
<td>0.03</td>
<td>4%</td>
<td>7%</td>
</tr>
</tbody>
</table>

Calculate the account value at the end of the second year.

**Solution:**

\[
AV_t = AV_{t-1} + P_t(1 - f) - EC_t - \left( \frac{ADB_t}{1 + i^e} \right) (Rate_t) (1 + i^e)
\]

\[
AV_1 = 0 + (10,000)(1 - 0.25) - 60 - \left( \frac{100,000}{1.04} \right) (0.02) (1.06) = 5847.94
\]

\[
AV_2 = 5847.94 + (12,500)(1 - 0.10) - 30 - \left( \frac{100,000}{1.04} \right) (0.03) (1.07) = 15,176.16
\]
3. A survivor whole life insurance pays a death benefit of 1 million at the end of the year of death of the second death of (50) and (60). The premiums are paid annually until the second death.

You are given:

a. Mortality follows the Illustrative Life Table.

b. Interest is at 6%.

Calculate the net annual benefit premium for this policy.

**Solution:**

\[
PVP = PVB
\]

\[
P\bar{a}_{50:60} = (1,000,000)A_{50:60}
\]

\[
P = \frac{(1,000,000)[A_{50} + A_{60} - A_{50:60}]}{\bar{a}_{50} + \bar{a}_{60} - \bar{a}_{50:60}} = \frac{(1,000,000)[0.24905 + 0.36913 - 0.42296]}{13.2668 + 11.1454 - 10.1944} = 13,730.68
\]
4. For a fully continuous whole life of 500,000 on (60), you are given:

   a. The gross premium reserve at $t = 20$ is 150,000.
   b. The gross premium reserve is estimated to be 154,444.72 at $t = 20.5$ using Euler’s method with $h = 0.5$.
   c. The gross premium is paid at a rate of $P$ per year during the 21st year.
   d. The force of interest is 5%.
   e. The force of mortality follows Makeham law with $A = 0.01$, $B = 0.002$ and $c = 1.04$.
   f. The following expenses payable continuously:
      i. 60% of premium in the first year and 10% of premium in years 2 and later;
      ii. 500 per policy in the first year and 30 per policy in years 2 and later; and
      iii. 1000 payable at the moment of death.

Calculate $P$.

Solution:

$$t+hV = tV + h\left[tV \cdot \delta_t + P_t(1 - f) - e_t + (DB_t + E_t - V) \mu_{x+t}\right]$$

154,444.72 = 150,000 +

$$0.5\left[(150,000)(0.05) + P(0.9) - 30 - (500,000 + 1000 - 150,000)(0.01 + 0.002(1.04)^{80} + 0.002\right]$$

$P = 23,456$
5. A fully discrete 3 year term pays a benefit of 10,000 upon any death. It pays an additional 5000 (for a total of 15,000) upon death from accident. It pays zero if the policyholder lapses. You are given:

<table>
<thead>
<tr>
<th>$x$</th>
<th>$q_x^{(1)}$</th>
<th>$q_x^{(2)}$</th>
<th>$q_x^{(3)}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>20</td>
<td>0.050</td>
<td>0.020</td>
<td>0.100</td>
</tr>
<tr>
<td>21</td>
<td>0.040</td>
<td>0.025</td>
<td>0.060</td>
</tr>
<tr>
<td>22</td>
<td>0.035</td>
<td>0.030</td>
<td>0</td>
</tr>
</tbody>
</table>

Decrement (1) is death from accidental causes. Decrement (2) is death from non-accidental causes. Finally, decrement (3) is lapse.

You are given that $d = 0.10$.

The level annual net premium for this insurance is 800 to the nearest 10. Calculate it to the nearest 1.

**Solution:**

\[
P = \frac{15,000 \left[ 5000v + 3320v^2 + 2541.875v^3 \right] + 10,000 \left[ 2000v + 2075v^2 + 2178.75v^3 \right]}{100,000 + 83,000v + 72,625v^2} = 797.87
\]

Calculate the net benefit reserve at the end of one year. (Note that the reserve could be negative.)

**Solution:**

Using the recursive formula:

\[
1V = \frac{[0 + P](1+i) - 15,000q_x^{(3)} - 10,000q_x^{(2)} - q_x^{(1)}}{p_x^{(r)}} = \\
[0 + 797.87] \frac{1}{0.9} - 15,000(0.05) - 10,000(0.02) - 0.9 \frac{1-0.05-0.02-0.10}{0.9} = -76.48
\]
6. A participating whole life insurance policy issued on (40). The policy pays a death benefit of 100,000 at the end of the year of death. You are given:

- Reserves are determined as net premium reserves using the net benefit reserve. The mortality follows the Illustrative Life Table and \( i = 0.06 \).
- The annual gross premium is 125% of the net benefit premium.
- Actual mortality experienced is 90% of the Illustrative Life Table.
- Actual interest earned is 7%.
- Expenses for the policy are:
  - Issue expenses of 200 per policy and 40% of premium.
  - Recurring expenses are 40 per policy and 8% of premium in all years including the first year.
- Cash values are 80% of the reserves.
- Withdrawals will occur at the end of the year. The rate of withdrawals will be 20% in the first year, 10% in the second year, and 7.5% per year thereafter.
- 60% of the profit each year will be distributed to the policyholders who were in force at the start of the year.

Calculate the dividend in for the 20th year.

Solution:

\[
P^x = (1.25) \frac{100,000 A_{40}}{\ddot{a}_{40}} = (1.25) \frac{(100,000)(0.16132)}{14.8166} = 1360.97
\]

\[
n_{19} V = (100,000) \left[ 1 - \frac{\ddot{a}_{99}}{\ddot{a}_{40}} \right] = (100,000) \left[ 1 - \frac{11.3818}{14.8166} \right] = 23,182.11
\]

\[
n_{20} V = (100,000) \left[ 1 - \frac{\ddot{a}_{60}}{\ddot{a}_{40}} \right] = (100,000) \left[ 1 - \frac{10.9041}{14.8166} \right] = 24,777.61
\]

\[
Pr = [23,182.11 + 1360.97(1 - 0.08) - 40](1.07) - (100,000)(0.9)(0.01262)
- (24,777.61)(0.8)[1 - (0.9)(0.01262)](0.075) - (24,777.61)[1 - (0.9)(0.01262)](1 - 0.075)
\]

\[
= 837.25
\]

\[
Div = (837.25)(0.6) = 502.33
\]
7. A special 4 year term insurance on (90) pays a death benefit of 100,000 for death in the first year. It pays 110,000 for death in the second year. It pays a death benefit of 120,000 for death in the third year. Finally, it pays a death benefit of 130,000 for death in the fourth year.

All death benefits are paid at the end of the year of death.

Level annual premiums are paid for the life of the insured.

You are given that $q_{90+x} = 0.1 + 0.1 \times r$.

You are also given the following spot interest rates:

<table>
<thead>
<tr>
<th>Term</th>
<th>Spot Interest Rates</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6%</td>
</tr>
<tr>
<td>2</td>
<td>7%</td>
</tr>
<tr>
<td>3</td>
<td>8%</td>
</tr>
<tr>
<td>4</td>
<td>9%</td>
</tr>
</tbody>
</table>

The net annual benefit premium is 22,900 to the nearest 100. Determine the premium to the nearest 1.

Solution:

\[
P = \frac{(100,000) \frac{100}{1.06} + (110,000) \frac{180}{(1.07)^2} + (120,000) \frac{216}{(1.08)^3} + (130,000) \frac{201.6}{(1.09)^4}}{1000 + \frac{900}{1.06} + \frac{720}{(1.07)^2} + \frac{504}{(1.08)^3}} = 22,887.44
\]

Calculate the net benefit reserve at the end of the second year of this policy.
Solution:

Using the recursive formula

\[
V_1 = \frac{(V + P)(1 + f_{[0,1]}) - B_1 \cdot q_{90}}{p_{90}} = \frac{(0 + 22,887.44)(1.06) - (100,000)(0.1)}{1 - 0.1} = 15,845.21
\]

\[
V_2 = \frac{(V + P)(1 + f_{[1,2]}) - B_2 \cdot q_{91}}{p_{91}} = \frac{(15,845.21 + 22,887.44)(1.07)^2(1.06) - (110,000)(0.2)}{1 - 0.2} = 24,793.65
\]

Note: \( f_{[0,1]} \) and \( f_{[1,2]} \) are the forward interest rates. These must be used. You cannot use the spot rates!