Chapter 12

1. You are given the following for a five year term on (50):

   a. The gross premium payable annually is 300.
   b. The death benefit is 32,000 payable at the end of the year of death.
   c. Mortality is the only decrement and follows the Illustrative Life Table.
   d. The interest rate earned will be 7%.
   e. The reserves in all years are zero.
   f. The issue expenses at the start of the first year are 180 per policy plus 5% of premium.
   g. The maintenance expenses at the start of all years including year 1 are 20 per policy and 5% of premium.

Calculate the profit vector for this policy.

**Solution:**

See Spreadsheet.

2. You are given the following for a five year term on (65):

   a. The gross premium payable annually is 300.
   b. The death benefit is 10,000 payable at the end of the year of death.
   c. Mortality is the only decrement and follows the Illustrative Life Table.
   d. The interest rate earned will be 6%.
   e. The profit at the end of the first year is 14.54.
   f. The issue expenses at the start of the first year are 80 per policy plus 5% of premium.
   g. The maintenance expenses at the start of all years are 20 per policy and 5% of premium.

Calculate the reserve per policy at the end of the first year.

**Solution:**

Profit = (BegReserve + Premium - Expense)(1+i) - Benefits - (EndReserve)(p^{(r)})

14.54 = (0 + 300 - 20 - (0.05)(300))(1.06) - (10,000)(0.02132) - (EndReserve)(1 - 0.02132)

EndReserve = \frac{(0 + 300 - 20 - (0.05)(300))(1.06) - (10,000)(0.02132) - 14.54}{1 - 0.02132} = 54.32
3. You are completing a profit test on a whole life issued to (60). Further, you are given:

   a. The gross premium payable annually is 3000.
   b. The expenses in the 10 policy year are 258.
   c. The interest rate used in the profit test is 5%.
   d. The reserve per policy at the end of the 9th policy year is 10,000.
   e. The reserve per policy at the end of the 10th policy year is 11,500.
   f. Mortality is the only decrement and follows the Illustrative Life Table.
   g. The profit in the 10th policy year is 200 for each policy that was in force at the start of the policy year.

Calculate the amount of the death benefit.

\[
\text{Profit} = \text{(BegReserve} + \text{Premium} - \text{Expense})(1+i) - \text{Benefits} - \text{(EndReserve)}(p^{(x)})
\]

\[
200 = (10,000 + 3000 - 258)(1.05) - (DB)(0.03037) - (11,500)(1 - 0.03037)
\]

\[
\text{EndReserve} = \frac{(10,000 + 3000 - 258)(1.05) - (11,500)(1 - 0.03037) - 200}{0.03037} = 66,788.11
\]
4. *For a fully discrete whole life policy on (50) with a death benefit of 100,000, you are given:

   a. Reserves equal benefit reserves calculated using the Illustrative Life Table at 6%.
   b. The gross premium equals 120% of the benefit premium calculated using the Illustrative Life Table at 6%.
   c. Expected expenses equal 40 plus 5% of gross premium, payable at the beginning of the year.
   d. Expected mortality equals 70% of the Illustrative Life Table.
   e. The expected interest rate is 7%.
   f. There are no withdrawals.

Calculate the expected profit in the eleventh policy year, for a policy in force at the beginning of the year.

Solution:

Profit = (BegReserve + Premium - Expense)(1+i) - Benefits - (EndReserve)(p^{(r)})

\[ P^{Net} = \frac{100,000 A_{sz0}}{\bar{a}_{sz0}} = \frac{(100,000)(0.24905)}{13.2668} = 1877.24244 \]
\[ P^{Gross} = (1.2)(1877.24244) = 2252.69 \]

\[ V_{10} = 100,000 \left(1 - \frac{\bar{a}_{sz0}}{\bar{a}_{sz0}}\right) = 100,000 \left(1 - \frac{11.1454}{13.2668}\right) = 15,990.29 \]

\[ V_{11} = 100,000 \left(1 - \frac{\bar{a}_{sz1}}{\bar{a}_{sz0}}\right) = 100,000 \left(1 - \frac{10.9041}{13.2668}\right) = 17,809.12 \]

Profit = (15,990.29 + (2252.69)(0.95) - 40)(1.07) - (100,000)(0.01376)(0.7) - (17,809.12)(1 - (0.01376)(0.7))

= 755.89
5. You are given the following profit signature:

<table>
<thead>
<tr>
<th>Time</th>
<th>Profit Signature</th>
<th>Premiums</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-200</td>
<td>400</td>
</tr>
<tr>
<td>1</td>
<td>40</td>
<td>350</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>300</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>250</td>
</tr>
<tr>
<td>4</td>
<td>60</td>
<td></td>
</tr>
</tbody>
</table>

a. Calculate the internal rate of return.

**Solution:**

Using BA II+
CF0=-200; C01=40; C02=100; C03=100; C04=60; IRR CPT = 17.4%

b. Calculate the net present value at 12%.

**Solution:**

\[
NPV = -200v^0 + 40v^1 + 100v^2 + 100v^3 + 60v^4 =
-200 + 40(1.12)^{-1} + 100(1.12)^{-2} + 100(1.12)^{-3} + 60(1.12)^{-4} = 24.74
\]

c. Calculate the discounted payback period at 8%.

**Solution:**

The discounted payback period is the smallest \( m \) such that \( NPV(m) > 0 \).
Looking at the profits, we know that the earliest that \( NPV(m) > 0 \) is for \( m = 3 \) since the sum of the profits must be > 0 for \( NPV(m) > 0 \). Try \( m = 3 \)
\[
NPV(3) = -200v^0 + 40v^1 + 100v^2 + 100v^3 =
-200 + 40(1.08)^{-1} + 100(1.08)^{-2} + 100(1.08)^{-3} = 2.15
\]
So Discounted Payback Period is 3 years.
d. Calculate the profit margin at 12%.

Solution: 

\[
\text{Profit Margin} = \frac{NPV}{\text{PV of Prem}}
\]

\[
\text{PV of Prem} = 400 + 350(1.12)^{-1} + 300(1.12)^{-2} + 250(1.12)^{-3} = 1129.60
\]

\[
\text{Profit Margin} = \frac{NPV}{\text{PV of Prem}} = \frac{24.74 \text{ From Part (b)}}{1129.60} = 2.19\%
\]

6. You are given the following profit vector for a policy issued to (x):

<table>
<thead>
<tr>
<th>Time</th>
<th>Profit Vector</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-200</td>
</tr>
<tr>
<td>1</td>
<td>40</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
</tr>
<tr>
<td>4</td>
<td>60</td>
</tr>
</tbody>
</table>

The mortality rate for is \( q_{x+t} = 0.03 + 0.01t \) for \( t = 0, 1, 2, 3 \).

There are no withdrawals.

Calculate the Expected Present Value of Future Profits at 10%.

Solution:

Expected Present Value of Profits = \( NPV = \)

\[
-200(0 p_x^{(r)})v^0 + 40(0 p_x^{(r)})v^1 + 100(1 p_x^{(r)})v^2 + 100(2 p_x^{(r)})v^3 + 60(3 p_x^{(r)})v^4 =
\]

\[
-200 + 40(1.10)^{-1} + 100(1 - 0.03)(1.10)^{-2} + 100(1 - 0.03)(1 - 0.04)(1.10)^{-3} + 60(1 - 0.03)(1 - 0.04)(1 - 0.05)(1.10)^{-4} = 22.74
\]
Answers

1. -195, 94.11, 78.11, 60.51, 40.99, 19.87
2. 54.32
3. 66,788.11
4. 756
5. 
   a. 17.4%
   b. 24.74
   c. 3 Years
   d. 2.19%
6. 22.74
7. 